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AN ANALYTIC MODEL OF THE STRATEGIC BOMBER PENETRATION MISSION W--ETC(U)
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MISSION WITH VARIANCE CALCULATIONS

THESIS

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THESIS

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by

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ABSTRACT

This paper describes an analytic model of the strategic bomber penetration mission. The model covers all parts of the mission from take off to kill or recovery. Specifically, the model includes representations of base escape, aerial refueling, forward defense, barrier SAMs, area fighter-intercepter defense, random area SAMs, terminal SAMs, weapons delivered, and recovery. Cruise missiles are included in the model.

For each part of the mission listed above, a distribution is derived for the number of bombers that survive given the number that begin that stage of the mission. A computer program has been written to convolute these distributions to find distributions for the total number of bombers that survive, the total number of weapons released, and the total number of cruise missiles that destroy targets.

One major advantage of this model is its scope. It includes descriptions of all the events that occur during the mission which will have an impact on the outcomes of later parts of the mission.

A second major advantage of this model is the inclusion of calculations for the variances of the results of the model. This is an improvement on other analytic models which give only expected value results and hence contain no indication of the possible variability of those results.

All the mathematical derivation for this model are contained in this paper. The paper also includes some initial results from the computer program which demonstrate some of the many outputs that can be obtained from the model.

CHAPTER I

INTRODUCTION

Models of the strategic bomber mission have always been of great interest to the Air Force. In recent years this interest has been heightened by contraversies, often of a political nature, that have developed. Much debate has centered on the need to replace the Air Force's aging fleet of B-52 bombers with a more modern weapon such as the B-1 or a derivative of it. The problems of a bomber fleet's survivability and its ability to penetrate a defended country raise the question of whether the manned bomber is still a viable part of the strategic deterrent when compared to such alternatives as cruise missiles or land and sea based ballistic missiles. If one concludes that a manned bomber can still be an effective strategic weapon, the question remains whether a new bomber could justify its cost by performing the mission significantly better than the B-52.

To help answer these and many other questions about the strategic bomber mission, numerous models have been developed which describe all or part of the mission. These models fall into two broad categories, computer simulation models and analytic models. In general, computer simulations can offer greater levels of detail and potentially greater flexibility; however, large simulations can be both costly and time consuming to run. Analytic models are usually cheaper and faster, but they generally give only expected value results which may not contain the same amount of information as a large number of runs of a simulation model. In particular, the analytic models seldom give any indi-

cation of the possible uncertainty (variability) of their results. This is the issue of primary concern in this paper.

The variance, or alternatively, the standard deivation, of a result from a model is an important measure. It gives an analyst some idea of the range of values being represented in his results. This in turn allows him to better evaluate the degree of precision contained in the results. This can help an analyst avoid overemphasizing the importance or misinterpreting the meaning of the results of his model. It can in some circumstances be an important factor in a decision involving a choice between alternatives.

As an example of how the variance of a result could affect a decision, consider the following situation. A small airstrike is being planned against a defended target. Two alternatives are available. Plan A is to send two aircraft against the target, each carrying one weapon. These aircraft are estimated to each have a 60-40 chance of penetrating the defense and reaching the target. Plan B involves a single newer aircraft having twice the payload of the older aircraft and an estimated 60-40 chance of surviving the defense and reaching the target.

In Plan A, if the two aircraft are independent of each other, the distribution for the number of weapons that reach the target is

P(2 weapons reach target) = 0.36

P(1 weapon reaches target) = 0.48

P(0 weapons reach target) = 0.16

Under Plan B, the single aircraft will either reach the target and release all its weapons or fail to reach the target at all. The distribution for the number of weapons reaching the target is

P(2 weapons reach target) = 0.6

P(0 weapons reach target) = 0.4

The expected number of weapons that would reach the target under either plan is 1.2. If this is the only information aviable to the decision maker, he will likely conclude that both plans are equally effective.

Now look at the variances of the two results. Plan A has a variance of 0.48 while Plan B has a variance of 0.96. If the decision maker's primary concern is to have at one weapon reach the target, then knowing the relative variances involved in the two plans he will likely choose Plan A instead of Plan B. Even though the expected total number of weapons reaching the target is the same in both plans, the smaller variance of Plan A indicates that it may actually have a better chance of delivering at least one weapon to the target. That this is indeed the fact can be verified from the distributions.

This is admittedly a very simple example but it serves nonetheless to illustrate the importance of knowing the variance in model results. In a simulation model, an estimate of the variance of the results can be obtained by running the model several times and calculating the sample variance of the results. An analytic model, however, generally gives only expected value results and no variance information obtained. This is the case in virtually all of the analytic models currently existing that deal with the strategic bomber penetration problem.

The original suggestion for a thesis in the area of strategic bomber penetration came from Captain R. Wilkinson at the Strategic Systems Program Office of the Aerospace Systems Division (Strat SPO).

Of current interest to the Strat SPO is an expected value analytic model of the bomber penetration problem developed by William J. Schultis at the Institute for Defense Analysis (Ref 13). Captain Wilkinson suggested that the Schultis model, which currently does not cover the problems of base escape, aerial refueling, and recovery, be expanded to include the entire strategic bomber mission. Because the Schultis model is an expected value model with no variance calculations, it was decided to broaden the scope of the project to include the variance problem. In doing this, some parts of the Schultis approach were employed; however, some concepts were added from other models and more were developed specifically for this model. The Schultis model, along with other models in the area of strategic bomber penetration, is discussed in more detail in the next chapter.

OBJECTIVES AND SCOPE

The purpose of this paper is to develop an analytic, national level model of the strategic bomber mission from take-off to kill or recovery. The object is to construct the model to allow for both expected value and variance calculations.

Structure. The strategic bomber mission can be divided up into nine parts, not all mutually exclusive, which define the major aspects of the mission. These are:

a) Base Escape - bombers escape from their own bases which may be under SLBM attack;

b) Aerial Refueling - bombers are refueled in flight during the mission;

c) Forward Air Defense - bombers are attacked by defensive interceptors before reaching the defended continent;

d) Barrier SAMs - bombers are attacked by a perimeter SAM defense as they reach the coast of the defended continent;

e) Random Area SAMs - bombers penetrate to targets inside an area defended by randomly located SAM sites;

f) Fighter-Interceptors - bombers are attacked by defensive fighter-interceptors while penetrating to their targets;

g) Weapons Delivered/Terminal SAMs - bombers deliver weapons at targets which may be defended by terminal SAMs;

h) Recovery - bombers leave the defended area and attempt to find a friendly base at which to recover;

i) Cruise Missiles (CMs) - during the mission, bombers may release CMs which will penetrate to targets on their own.

Philosophy. Because of the size and complexity of this model of the bomber penetration problem, the philosophy of this model is to sacrifice resolution where necessary in favor of breadth. In the final chapter of this paper some areas are suggested where it should be possible to build further detail into the model. However, it should be recognized that there are practical limitations to the level of detail that can be obtained in an analytic model of this scope. To achieve finer detail, it will likely be necessary to return to large simulation models such as the Boeing Company's Advanced Peneteration Model (see Chapter II).

APPROACH

The approach used to model the strategic bomber mission is to first separate the mission into parts that can be modelled individually and then to integrate the various parts into a complete model. In

terms of the parts of the mission previously defined, it will be assumed that the first four parts, base escape, aerial refueling, forward air defense, and barrier SAMs, occur sequentially and do not overlap. Hence, they can each be modelled separately. Random Area SAM and figher-interceptor defenses, however, occur simultaneously. In this model, they are assumed to operate independently of each other and so are initially considered separately. The results of each are then combined in the model under area defenses. The terminal SAM and weapons delivered part of the mission would in an actual mission overlap the area defense stage; however, in this model it will be treated separately. Recovery is the last part of the mission and will be treated separately in the model. The release of curise missiles will be considered at three different points in the mission; first between the refueling and FAD stages; second, between the FAD and BSAM stages; and finally, during the area defense penetration stage.

A flow chart of the model is shown in Figure 1.1. Each part of the model will be developed independently of the others and the outputs from early parts will be the inputs to later parts. An outline of the various parts of the model is given below.

Base Escape. The initial number of bombers in the problem is an input and the model finds a distribution for the number of bombers that survive the SLBM attack on their bases.

Aerial Refueling. The initial number of tankers is an input and the model finds a distribution of the number of bombers that successfully refuel for a given number that survived the base escape.

Forward Air Defense. The number of defensive interceptors is an input and the model finds distributions for the number of bombers

that survive the FAD given the number that escaped their bases and the number that were refueled.

Barrier SAMs. The number of SAM sites in the perimeter defense and the width of the perimeter are input and the model gives distributions for the number of bombers that survive the BSAM defense given the number that survived the FAD.

Area Defense. The number of SAM sites in the defended area and the size of the defended area are input. The model finds a single bomber's probability of penetrating through the RASAM defenses as a function of the depth of penetration.

An intercept intensity parameter describing the average number of intercepts that could occur on a detected bomber per unit time is input and the model finds a bomber's probability of penetrating through area FI defenses as a function of the depth of penetration.

The two penetration probabilities are then combined to give a single probability that a bomber survives the area defenses as a function of the depth of penetration.

Terminal SAMs and Weapons Delivered. The bombers' target set and the level of terminal defenses at the targets are input and the model gives a distribution for the total number of weapons delivered by a given number of bombers that have survived the BSAM defense.

Recovery. The model gives a distribution for the number of bombers that survive to exit the defended area for a given number of bombers that have survived the BSAM defense.

Cruise Missiles. The number of CMs on each bomber and a release policy are input and the model gives a distribution for the total num-

ber of CMs that reach their targets. Cruise missiles may be released at any of three points in the model.

In this model, the defended area in which the targets lie is assumed to be a rectangular corridor. The BSAM defenses lie at the northern boundary of the corridor and bombers enter from the north and penetrate to their targets along straight lines to the south. Once the bombers have released their weapons they do not attempt to return to the northern boundary but continue to fly south until they exit the corridor at the southern end.

FORMAT

Chapter II of this paper gives a brief review of some of the currently existing models that deal with the strategic bomber mission.

Chapter III presents a more detailed description of the stages of the bomber mission to be modelled. The major assumptions made in the model are discussed in this chapter.

Chapter IV develops the model representation of the first four stages of the strategic bomber mission. These are base escape, aerial refueling, forward air defense, and the barrier SAM defense.

Chapter V develops model representations for the two forms of area defense, random area SAMs and fighter-interceptors.

Chapter VI develops model representations for the final parts of the strategic bomber mission, terminal SAMs and weapons delivered, and recovery. This chapter also includes the modelling of cruise missiles.

Chapter VII shows how the various parts of the model that have been developed in the previous chapters are integrated to form a complete model. The chapter includes a brief discussion of the computerization of the model.

Chapter VIII briefly discusses verification of the model and shows some of the model results.

Chapter IX presents a summary of this paper along with some conclusions and recommendations.

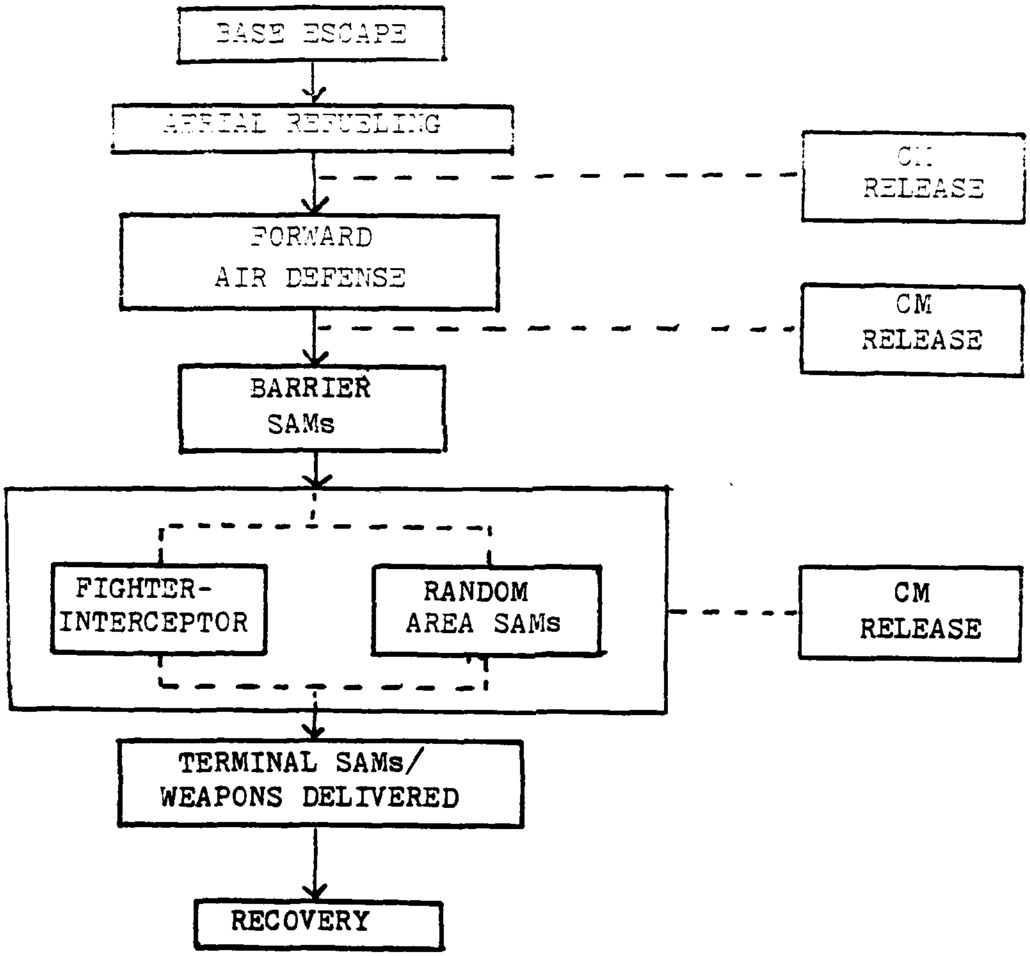

FIGURE 1.1

Flowchart of the model.

CHAPTER II

REVIEW OF CURRENT MODELS

The purpose of this chapter is to briefly review some of the currently existing models, both analytic and simulation, that deal with all or parts of the strategic bomber penetration mission. Some of the models mentioned below are force on force models that treat missile exchange in addition to bomber penetration; they are included because they help to put in perspective the penetrating bomber's role in the strategic triad.

Simulations

Advanced Penetration Model. Perhaps the most comprehensive penetration model developed to date is the Advanced Penetration Model (APM), developed by Boeing for Headquarters USAF, the Assistant Chief of Staff, Studies and Analysis (Ref 11). The model simulates the strategic mission of the entire bomber and tanker force from takeoff to recovery (Ref 7:IV-4).

The model consists of two main parts, a Mission Planner and an Air Battle Simulator. The overall mission scenario is user defined; the model will then generate individual flight plans for each bomber (and tanker) in the force. Various rules or constraints may be imposed on the Mission Planner. The plan for each sortie will include routing, refueling, target allocation, and recovery (Ref 7:IV-8).

The Air Battle Simulation is a time sequenced process of the events that have been planned in the Mission Planner and include both deterministic and probabalistic events. Seven different processors are used to model the interaction of bombers and defenses over the

enemy's defended airspace. These are

a) Command and Control - assigns interceptors to penetrators;

b) Radar - detects penetrators; includes effects of ECM use;

c) AWACS;

d) Interceptor - vectors assigned interceptor to radar range of the penetrator, finds probability of detection and conversion and proability of kill and determines if the penetrator is killed;

e) SAM - finds probability of kill on a penetrator for a SAM and determines if the penetrator is killed;

f) Nuclear Effects - simulates CEP of a weapon and calculates the damage done by the weapon; and

g) Penetrator (Ref 7:IV-20).

All the results of the air battle simulations are stored as output, allowing a vast amount of specific information to be obtained from the model. The user may specify the particular parts of this information he wishes to analyze.

This wealth of output data and the relatively fine detail of the APM are its major advantages. The model can capture the complex realities of defense saturation, command and control limitations and weapon assignment policies (Ref 7:IV-4). At the same time, the vast amount of data that is inputted to the model offers great flexibility to the user.

However, in the ten-year history of the APM, this same level of detail and data handling has also been the model's single biggest drawback. Single replications of the model for any reasonably large force could require over twenty hours of computer time (Ref 7:IV-26). Extensive studies including repeated runs for sensitivity tests are

both time consuming and expensive. New generation computers, such as the IBM-3032 currently being used for the APM, have dramatically reduced the model's run time; however, the inputting and maintenance of the computer routines and data base remain costly and time consuming and prohibit widespread practical use of the APM (Ref 7:IV-26).

SPEED. SPEED (Simulation of Penetrator Encountering Extensive Defenses) was developed by Calspan Corporation and is a large Monte Carlo simulation of bomber penetration through air defense systems. SPEED simulates fighters, SAMs, C^2, ECM, and ground controlled intercepts (GCI) (Ref 8:13). SPEED is compatible with and complements the APM; in particular, the SPEED model can use the APM Mission Planner (Ref 1:B-12, 8:13).

FISCHER. FISCHER is an event oriented Monte Carlo simulation of an engagement between an attacking bomber force and a strategic defense system. The model was developed at Headquarters NORAD (Ref 1:B-6).

The model simulates the movements of bombers and air-to-surface weapons and their interaction with a defense system including both ground and airborne radars and interceptors which can be committed from either bases on CAP orbits. The model permits fuel monitoring, with reattack and recommit logic available (Ref 1:B-4). The original model has been modified for use by AF/SAS (Ref 1:B-4).

ANALYTIC MODELS

ARSENAL EXCHANGE MODEL. The Arsenal Exchange Model was developed by the Martin-Marietta Corporation. It is a large expected value model used to study the structure of total strategic forces (Ref 8:13). This model includes a perimeter area defense with a probability of

bomber penetration P_{pB} given by

$$P_{pB} = (1-P_E) + P_E(1-P_{AI})^{I/B}$$

where

P_E - probability of encountering the parameter defense

P_{AI} - probability that a bomber is killed by single interceptor pass

I,B - total number of interceptors and bombers respectively (Ref 11:5).

Several different techniques are used to optimize weapon allocation and the model can treat full force allocations, a variety of defenses and force design problems. A number of scenarios ranging from a single strike against military and value targets to three strike games involving problems of selecting a weapons reserve or a value target reserve for the initiaters third strike may be analyzed using the model (Ref 1:b-4). Three counterforce exchange options are also available including limited one- and two-strike models and an all out counterforce model (Ref 1:B-4). The model is used at Headquarters USAF, Studies and Analysis (AF/SA).

CODE-50. This is a widely used aggregated model developed by the Lambda Corporation (Ref 11:4). It is capable of handling a mixture of offensive weapons types. Bomber penetration probabilities which describe a bomber's chance of surviving against a specific type of fighter can be user input using data from other models. Alternatively, for B bombers and F fighters, the bomber penetration probability can be assumed to be proportional to $e^{-a(F/B)^c}$ where a and c are model inputs that describe each bomber/fighter pair and incorporate the

effects of most of the parameters effecting bomber penetration (Ref 11:4). The model is probably oversimplified in that it is difficult to find appropriate values for a and c that will adequately represent penetration probabilities generated by more detailed simulations or extracted from actual tactical exercises (Ref 11:5).

COLLIDE. COLLIDE is an aggregated conversion model for air combat designed to assess the impact of command and control on fighter-bomber engagements. The principle output of the model is a probability for interceptor detection and conversion under different engagement scenarios (Ref 1:B-5). The COLLIDE model incorporates bomber electronic counter measures (ECM) in its calculations. The COLLIDE model is used to estimate ECM effectiveness for the APM. Output from COLLIDE is used as input for the APM (Ref 6:IV-27).

COPEM-1 (Corridor Penetration Model). COPEM-1 was developed at Stanford Research Institute (SRI) as part of a study to improve the representation of airborne strategic systems in aggregated effectiveness evaluation models (Ref 12:v). The other part of the study was the development of an allocation model to use the output of COPEM-1.

COPEM-1 is a time dependent engagement model. Its purpose is to generate average bomber penetration probabilities as a function of the depth of penetration along a single attack corridor into a defended area. The model includes only fighter area defenses; SAMs are handled separately in the allocation model (Ref 5:5). The underlying assumption in the model is that the number of intercepts that occur on a bomber follow a Poisson distribution with a time dependent parameter (Ref 5:6). This parameter is calculated iteratively at discrete time

intervals during the engagement. It incorporates all the information about the defensive fighter except for the single pass kill probability for a fighter against a bomber which is a separate input (Ref 5:6). The basic concepts of the COPEM-1 model have been employed in this paper to model bomber penetration of area fighter defenses. The mathematical development is presented in Chapter V. Later versions of this model have gone by the name BOMPEN; the names are frequently used interchangeably (Ref 11:20).

HISTVEC. This is a fast running expected value model of bomber penetration. Fighters and bomber decoys are both modelled in detail (Ref 8:13). In particular, the model considers fighter airbase deployment, different fighter types with different AAMs possible and detection and conversion probabilities that are functions of both fighter type and penetration altitude (Ref 8:15). Decoy considerations such as flight range, credibility, threat dilution, and primary payload displacement are all incorporated in the model (Ref 8:16). The model includes no representations of BDMs or SAM defenses and does not model ECM (Ref 8:16,19).

LULEJIAN-MARKOVIAN. A model developed by Lulejian and Associates uses a Markov process to model the bomber penetration problem (Ref 8:13). Both fighters and decoys are modelled in as great detail as in HISTVEC with the added feature that fighters can be reassigned while airborne (Ref 8:15). SAM systems are modelled with two two different SAM types allowed. Firing rate limits and degradation by chaff or ECM are also included in the model's representation of SAMs. BDMs are modelled in detail including the option of short- or

long-range BDMs. The target structure can also be modelled. The effects of a precursor ICBM/SLBM strike can be included in the model (Ref 8:15-16).

PENETRATION INTEGRALS. The penetration integrals were formulated by Carl Builder at the RAND Corporation (Ref 15:1). The basic approach is to use differential equations to model combat based on random encounters between interceptors and penetrators. Builder's equations give simple analytic results for the extreme cases where interceptors have unlimited weapons load or unlimited attack time available (Ref 15:1). Builder's penetration integral unifies these two extremes and gives a general solution for the cases where there are restrictions on weapons loads and/or attack time (Ref 15:1, 4:iii).

PENEX. PENEX models strategic bomber penetration against manned interceptors (SAM defenses are not included). The basic assumption made is that the air battle can be divided up into discrete "sub-battles" which occur sequentially in time. Penetrators and decoys that survive one sub-battle will progress to the next. At each new sub-battle, fresh interceptors may be added to those that have capability remaining after previous sub-battles. Both close controlled and raid controlled interceptors are modelled. The basic model has been extended to consider several types of interceptors and to consider the effect of command and control system confusion resulting in false tracks or failures to track some penetrators. The model was developed at AF/SA (Ref 2).

THE SCHULTIS MODEL. Next a model which had a substantial impact on this thesis is discussed. The Schultis model, titled "A National-Level Analytic Model for Penetration of Various Combined Air Defense Deployments by Cruise Missiles or Bombers," is a small (capable of being run

on a TI-59) expected value penetration model (Ref 13). Five types of air defenses are modelled; forward air defense, barrier SAMs, random area SAMs, fighter-interceptors, and terminal SAMs. The model uses value extracted as its measure of effectiveness (Ref 13:2).

The basic approach taken in the model is to separate the defenses into bands that are penetrated sequentially by the bombers. In this approach, the FI defense is envisioned as a band which lies inside the barrier SAMs but does not actually include the heartland where the targets lie. Random area SAMs are interspersed with the targets but do not interfere with the FIs.

The model deals with large numbers of penetrators, particularly CMs, and relies on saturation of the defenses, rather than leakage, as the primary method of penetration. For this reason, it is assumed that the offensive penetrators' best strategy is to attack in files along narrow corridors instead of individually at random. As the files approach each line of defense, losses may be expected at first; however, the SAMs will exhaust their missiles or the fighters within range of a particular file will exhaust their AAMs and in effect a path through any particular layer of defense will be cleared for the remaining penetrators.

Terminal SAMs are treated as a fixed price per unit value that must be paid by the penetrators to destroy a target of given value. Using this approach, the model can find the total value extracted by a given number of penetrators that have survived the bands of defenses for various allocation schemes (Ref 13).

As mentioned in the last chapter, it was the Schultis model that originally led to this project. In this paper, the same five basic

air defenses are modelled and to some extent, the concept of layers of defenses is employed. Unlike the Schultis model, however, this thesis does not treat fighter-interceptors as a band defense like the barrier SAMs. In the Schultis model, the two defenses are treated exactly the same way to the point that airbases are effectively modelled as SAM sites with wide coverages and individual FIs are modelled as SAMs which can kill more than one target.

The Schultis approach to modelling one-on-one penetrator/SAM engagements is used in this thesis; the mathematics are presented in Chapter IV in the secion on barrier SAMs. The concept of files, however, is replaced by a random attack and saturation of the defenses is not considered.

OTHER MODELS

The above are some of the more important and well-known models that have been developed in the area of strategic bomber penetration. The following is a partial listing of other penetration models that may be of interest.

a) Beta Cadens - a very large simulation model that includes all strategic forces and detailed damage assessment information (Ref 8:13).

b) NYLAND (RAND) - a small expected value model that includes bombers, decoys, BDMs, interceptors and SAMs (Ref 8:13-16).

c) OMEGA (SRI) - an allocation model that optimizes bomber force allocations against targets that may be locally defended by SAMs under a fighter umbrella (Ref 11:2).

d) PEGASOS (Academy for Interscience Methodology) - an expected value model which includes FI and SAM defenses, bomber decoys, and ICBMs. Does not cover BDMs (Ref 8:13-16).

e) QUICK (Lambda Corp.) - a large simulation that generates warplans and will optimize weapon allocations within restraints (Ref 11:7).

f) STRAT CROW - a computer simulation to model the effects of various types of ECM measures (Ref 1:B-15).

g) STRAT SPLASH - a model used to estimate single shot kill probabilities for AAMs (Ref 1:B-22).

h) STRAT SURVIVOR - a detailed simulation model of the base escape problem (Ref 1:B-23).

i) UNCLE (RAND) - a model specifically designed to account for the large number of uncertainties the offense and defense may have about each other (Ref 11:6).

This is by no means a complete list of the models that currently exist to describe the strategic bomber penetration mission. It does, however, cover most of the more important models. Some of these models, such as COLLIDE and STRAT SPLASH, may be useful when choosing values for some of the inputs of the model described in this paper.

CHAPTER III

PROBLEM DESCRIPTION AND ASSUMPTIONS

This chapter has two main purposes. The first purpose is to describe the basic parts of the mission of the strategic bomber force. The second purpose is to present some of the assumptions concerning the mission that will be made in this model.

An overview of the strategic bomber mission is shown in Figure 3.1. The bombers must first escape from their bases which are assumed to be under SLBM attack at the outset of a war with the USSR. At some point enroute to the USSR, the bombers will receive aerial refueling from tankers which have escaped from their own bases when the SLBM attack was launched. The bombers will then encounter a forward air defense system before reaching the northern coast of the USSR and a perimeter SAM defense immediately after reaching the coast.

If the bomber survives to this point, it will then penetrate to its targets through area defenses which may consist of SAMs and fighter-interceptors. At each target, the bomber may encounter terminal SAM defenses. Finally, after releasing all its weapons, the bomber will attempt to leave the USSR and recover at a friendly base.

The mission which has been outlined above can be divided into eight separate stages. These are a) base escape, b) aerial refueling, c) forward air defense, d) barrier SAMs, e) random area SAMs, f) fighter-interceptors, g) weapons delivered and terminal SAMs, and h) recovery.

In the remainder of this chapter, each of those stages will be described in greater detail and the major assumptions associated with each stage will be discussed. The last section of the chapter will discuss cruise missiles and the assumptions made by this model concerning them.

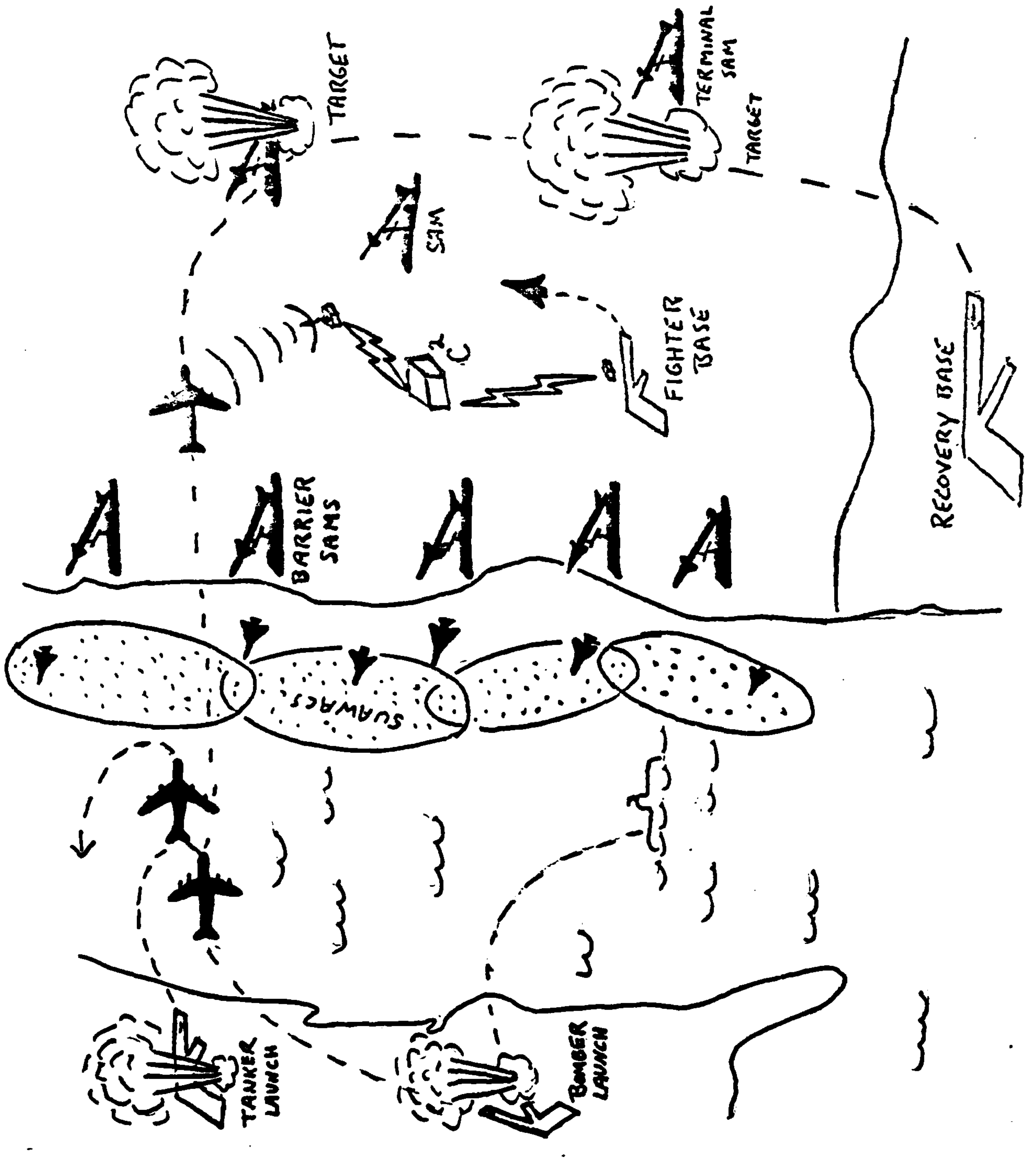

FIGURE 3.1

The strategic bomber penetration mission.

BASE ESCAPE

It is a commonly accepted assumption that a preemptive Soviet nuclear strike against the U.S. would include a SLBM attack on U.S. strategic bomber bases. The bombers' ability to escape the base and avoid the blast effects of the Soviet weapons will depend largely on the amount of warning they receive of the attack, their reaction time, and the number, size, and placement of the warheads targeted on the base.

At the time of the attack, some part of the bomber force stationed at any base may be on airborne alert. The number of bombers on airborne alert will depend greatly on the degree of perceived crisis immediately prior to the attack. For example, increased tension between the US and USSR during another war in the Middle East could be expected to lead to an increase in the number of bombers on airborne alert. The situation immediately prior to attack is shown in Figure 3.2.

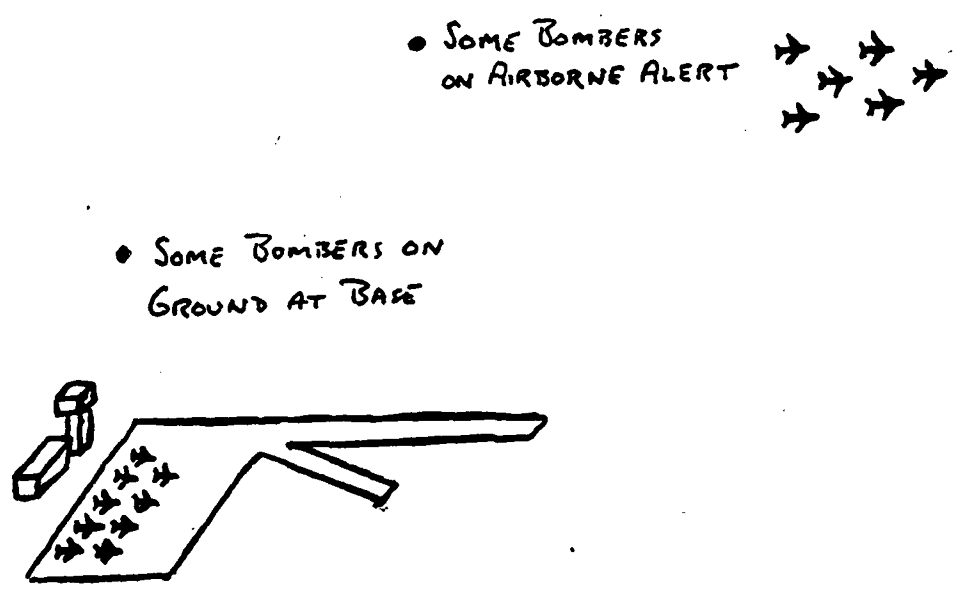

FIGURE 3.2

Situation at base prior to SLBM attack.

When the first warning of the attack is received, the bombers on the ground will begin to take off and dash away from the immediate vicinity of the base. The number of bombers that can get airborne will depend on the amount of warning they receive, their reaction time, and the rate at which they can take off from the base. Bombers that take off between the time the first warning of the attack is received and the time the warheads arrive and detonate are not guaranteed of successfully escaping. Should a bomber fly too close to a detonating warhead, it may still be destroyed. The pattern of warhead detonations around the base will depend on how each weapon is targeted and its CEP around its aim point. The situation is depicted in Figure 3.3.

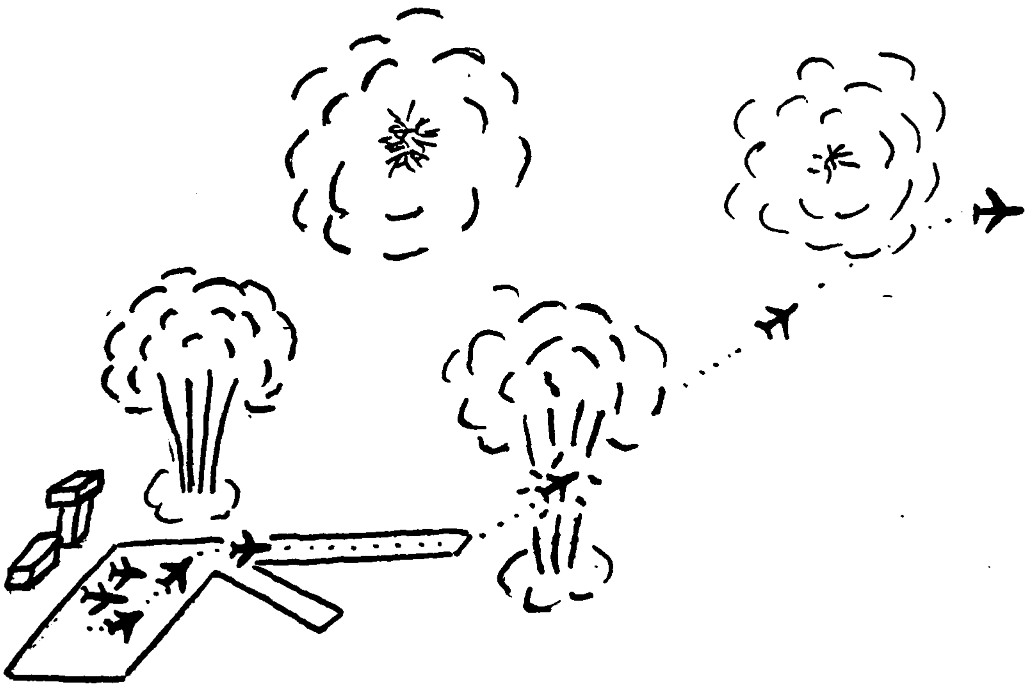

FIGURE 3.3

Situation at base during SLBM attack.

This model makes the following assumptions about the base escape problem. First, it is assumed that all bombers on airborne alert at the time of the attack are far enough away to be immune and are therefore assured of surviving the attack. All bombers not on airborne alert are assumed to be on the ground at the base when warning of the attack is first received.

To avoid the problems of warhead targeting and the location uncertainty of each warhead due to its CEP, this model assumes that the detonating warheads will be located randomly in the airspace around the base. However, it is further assumed that at least one warhead detonates close enough to the base to destroy all bombers remaining on the ground at that time.

REFUELING

Bombers will require airborne refueling to effectively accomplish their mission as planned. Refueling requirements depend on the bombers' payload, fuel capacity, and characteristics of its flight plan. Another factor is the amount of time a bomber may have spent on airborne alert prior to the enemy attack; he may not be able to complete his mission on the fuel load he has remaining.

Altitude is one important aspect of the bomber's flight plan that affects his fuel requirements. At low altitude, a bomber stands a better change of escaping detection by most radars and may have a better chance of surviving an attack by either a fighter interceptor or a SAM site. This is particularly true in the case of many older fighters which do not possess the so-called "look-down, shoot-down" capability. Hence, at some time during the mission a bomber would be expected to go to low altitude.

Unfortunately, at low altitudes, fuel consumption is much higher. In order to penetrate great distances at low altitude the bomber must begin the penetration part of its mission with a large fuel load. This necessitates airborne refueling at some time before encountering the enemy defenses.

With the current bomber force, it may often be the case that multiple refuelings are required. This will become especially true when cruise missiles are added to the bombers' payloads. Although requirements will vary among individual bombers with different mission plans, a typical bomber may expect to refuel twice in flight before reaching the first line of enemy defense. Each missile refueling will potentially degrade a bomber's effectiveness.

Successful refueling by a bomber depends on two main issues. The first issue is the availability of a tanker from which to refuel. Tankers that are based at the same bases as the bombers will face the same problem of base escape. This leaves some uncertainty in the exact number of tankers available to refuel bombers. The number of bombers each tanker can refuel will depend on the mission plan and the fuel requirements of each bomber being refueled. Typically, a tanker could expect to refuel two bombers one time each.

The second issue is that of a bomber's ability to locate and couple with an available tanker to be refueled. Detailed plans and alternate plans exist for each bomber specifying when and where to go to get refueled. However, navigational error enroute to the refueling point or equipment failure on the part of either bomber or tanker could prevent a successful refueling. The refueling problem is pictured in Figure 3.4. Because of the complexity of the problem, several assumptions are made for the purposes of this model.

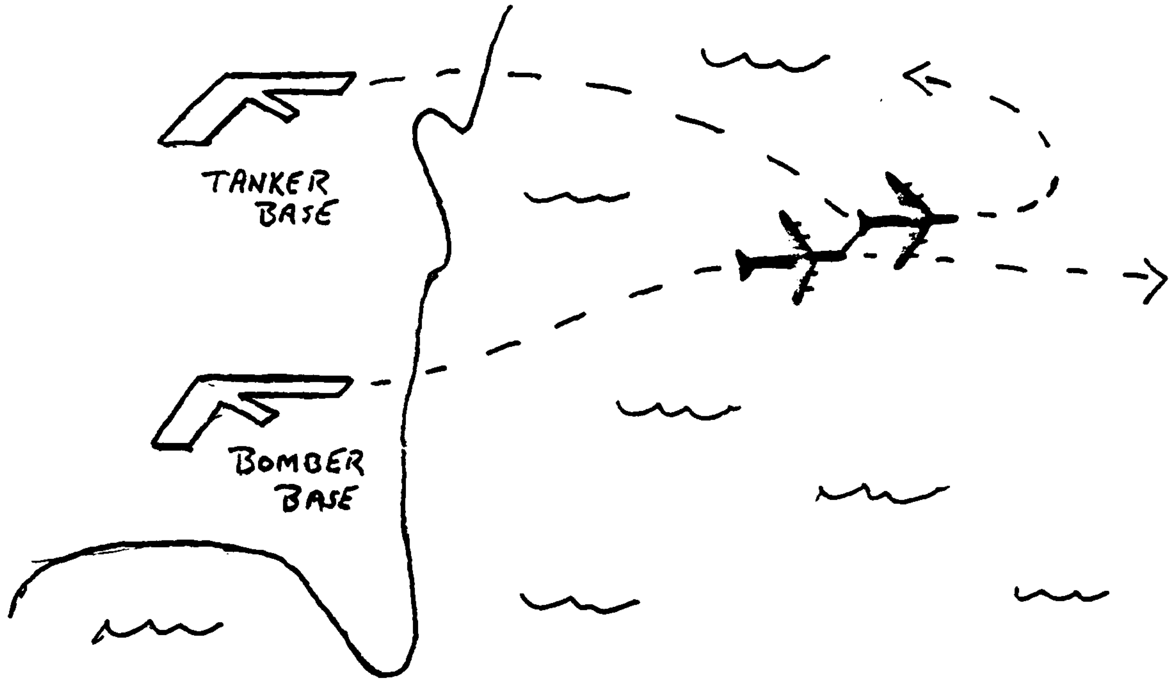

FIGURE 3.4

Aerial Refueling.

The first assumption deals with the number of tankers required to serve each bomber. Since on the average, bombers require two refuelings apiece and tankers are capable of two refuelings, the assumption is made in this model that one tanker is required to refuel each bomber. This does not imply that the bomber refuels twice from the same tanker, which is unlikely, only that the equivalent of one tanker load of fuel is necessary to refuel the bomber twice.

The next assumption made concerns a bomber's probability of successfully refueling, given that there is an available tanker. As long as a tanker is airborne for the bomber to refuel from, there will be a certain probability that it can successfully refuel once the tanker is found. The first probability could be estimated from the results

of numerous simulated missions flown each year and may be quite close to 1. The second probability could be obtained from equipment reliability data, perhaps adjusted to allow for damage that may occur to tankers during base escape. Together these two probabilities define a single probability of a bomber successfully completing one refueling. The second refueling will involve a different tanker and take place much later in the mission and the same process will be repeated. If the same probabilities of locating and successfully refueling apply, then the total probability of refueling twice will be the square of the probability of refueling the first time.

Since the bombers and tankers are following detailed plans, they are essentially matched bomber to tanker and are not searching randomly for each other. This assures that when a bomber locates its assigned tanker there will be fuel available for it; the tanker will not have already off-loaded its fuel onto other bombers.

The above reasoning leads to the assumption that a bomber's probability of successfully refueling, given an available tanker, can be summed up in one number which takes into account finding the tanker, successfully refueling from the tanker, and repeating the process later in the mission. This probability will be constant for all bombers and each bomber's refueling event will be independent of the other bombers'.

The final assumption deals with what happens to bombers that fail to get refueled. In general, a bomber that does not get refueled will continue its mission. It will still be capable of reaching targets within the Soviet territory. It may, however, be forced to fly much or all of its mission at high altitude where it is more easily detected

and attacked by Soviet defenses but where its fuel consumption rate will be much lower.

For the purposes of this model, it will be assumed that all bombers that fail to refuel will continue their missions but will penetrate at high altitude while bombers that successfully refuel will choose to penetrate at low altitude. After the refueling stage of the mission, the model will treat the refueled and unrefueled bombers as separate and essentially independent groups for the remainder of the mission. Also, this model does not consider the case where a bomber receives only one refueling; bombers are assumed to either successfully refuel twice or fail to refuel.

FORWARD AIR DEFENSE

The first line of defense the bombers will meet is the forward air defense (FAD). FAD systems attempt to attack the bombers before they reach the defended continent and have a chance to release their weapons.

A FAD system will include two major components. One is some system of warning and possibly detection of approaching bombers. The other component will be a force of fighter-interceptors sent out to engage the bombers.

The system will operate in one of two general modes. In the close control mode, a sensor or sensor network is used to detect and track the incoming bombers. The fighters are then uniformly assigned to the bombers by a control system and vectored to appropriate intercept points. Both detection and control systems may be part of an airborne warning and control system (AWACS). An AWACS system could contain radars capable of looking both out and down and would be able to detect both high and low flying bombers.

The second mode in which a FAD system could operate is the raid control mode. In this case, warning is received of the approaching bombers and the defense will commit interceptors to an airborne search over a large area. Bombers are engaged as they are located and the interceptors continue the search while fuel and ammunition remain.

This model assumes a FAD system composed of early warning and control aircraft (SUAWACS) and a fighter force which is close controlled as in Figure 3.5. It is assumed that there exist sufficient SUAWACS to form a perimeter of overlapping coverages so that bombers cannot leak through undetected. It is assumed that the SUAWACS are capable of making a highly accurate count of the incoming bombers.

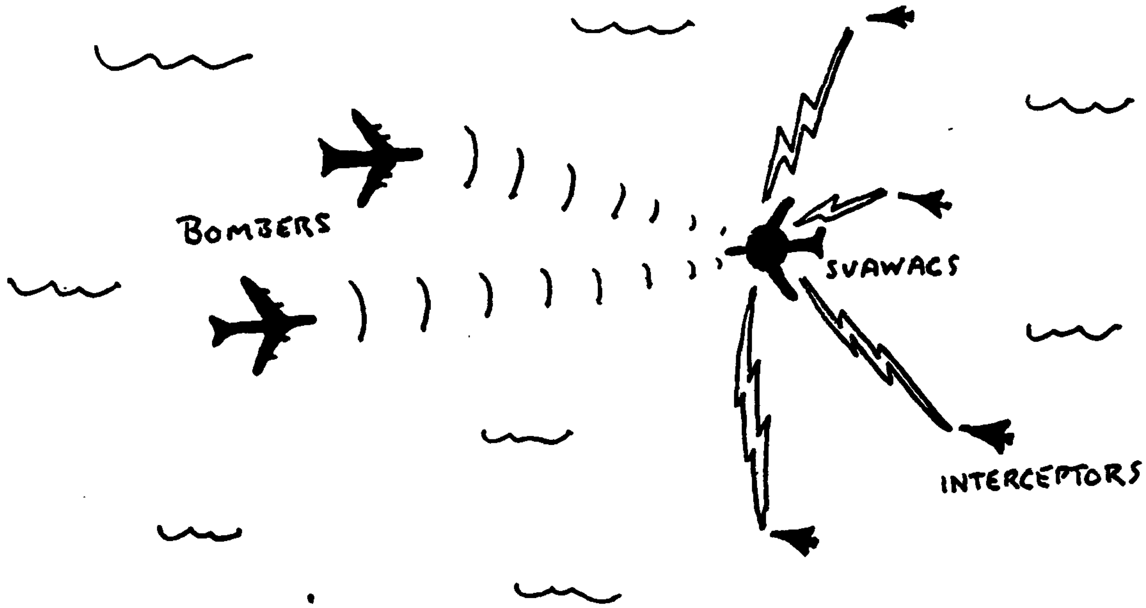

FIGURE 3.5

Forward Air Defense.

Another assumption that is made is that each fighter can attack at most one bomber. In support of this assumption, consider the following scenario.

A fighter takes off from its base and flies to its assigned CAP position where it may orbit for a while until a bomber is detected. The fighter is then assigned to the bomber and vectored by the SUAWACS to its interception point. Having arrived there, the fighter perhaps has to search briefly before locating the bomber. Once the bomber is located, the FI attempts to convert on and kill it. After the attack, even if the fighter still has some weapons remaining, fuel must be a consideration. If the approaching bombers are separated by distances large enough to prevent the fighter from immediately locating a second target, the fighter will probably have to return to base. By the time he has returned, landed and refueled, the FAD battle will have ended. (The fighter may, however, still return to fight as part of the area FI defenses of the heartland.) Note that the above scenario implicitly assumes that the defensive fighters do not have an aerial refueling capability.

<u>BARRIER SAMs</u>

After leaving the FAD zone, the bombers will approach the defended continent. Shortly after reaching the coast, they may encounter a barrier or band of SAM sites (BSAM). The coverage of this barrier may not extend too deep into the defended area, but the barrier would presumably be as wide as possible. This will present the penetrating bombers with a wall of defenses which they cannot go around and must therefore break through before penetrating into the Soviet heartland.

Ideally, the defense would like to place the SAM sites so that their coverages overlapped at all altitudes, making the band uniformly defended. However, the defense may be limited in the total number of SAMs he has available to deploy and the location of the sites he has will be effected by terrain considerations. In practice therefore, the band may not be uniformly defended; in some places site coverages overlap while in other places there may exist gaps in the defense. The situation is shown in Figure 3.6.

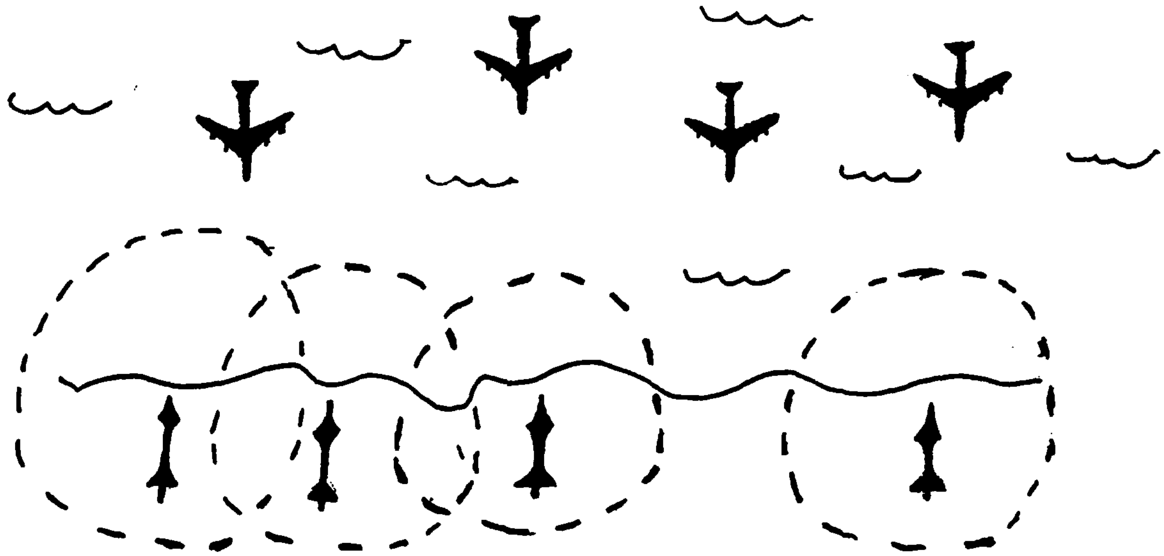

FIGURE 3.6

Barrier SAM defense.

If the offense knew with reasonable certainty before the attack where the SAM sites were located, the bombers could navigate courses through the gaps in the barrier. Alternatively, they could employ ECM to degrade the effective radius of a site or suppress the site entirely

with a SRAM. Any of these measures would allow the offense to reduce or avoid altogether losses to the BSAMs.

Unfortunately, it is not possible for the offense to know with any certainty the location of SAMs prior to the attack. Most of the Soviet SAM systems are mobile. The missiles can be both transported and launched from tracked or even, as in the case of the SA-8, amphibious vehicles, while target acquisition and fire control radars can be deployed on other vehicles. This allows an entire firing battery to move at will around the country. Furthermore, this capability is not limited to SAMs of small effective radius. The Mobile SA-4, for example, has a range of 36 nautical miles and can reach targets at altitudes of 80,000 feet. Other SAM features include guidance systems designed to frustrate ECM measures. The aforementioned SA-8 employs an electro-optical tracker in conjunction with two separate guidance radars which allow it to launch two missiles at a target on two different guidance frequencies (Ref 9).

The model makes the following assumptions about the BSAM defense. First, it is assumed that the offense avoids or suppresses any SAM sites of which he is certain of the location prior to the attack. This implies that the only SAMs that will effect the problem are mobile SAMs which, because they can change position, cannot be briefed before the mission.

The model also assumes a homogenous SAM force. Although the different types of SAMs have varying effective ranges and altitudes, for this model they are all assumed to be described by the same average capabilities.

The final assumption concerns the placement of the mobile SAMs in the barrier. It is assumed that the defense cannot achieve the ideal deployment of overlapping SAM coverages for the entire width of the barrier, either because he is limited in resources or because terrain prevents him. However, if the SAMs are mobile, the offense will not know where the gaps are at any given time. Still, if the offense knows some points along the barrier are less likely to be defended than others, then he will presumably choose to attack through those points. Therefore, in order for the barrier to be most effective, the defense will wish to defend all parts of it as uniformly as possible. Thus, this model assumes that at any given time, the locations of the SAMs are distributed uniformly along the entire width of the barrier.

RANDOM AREA SAMs

After the bombers break through the barrier SAMs, they begin to penetrate deeper into the defended territory enroute to their targets. Along the way they may encounter two different forms of area defenses, random area SAMs (RASAMs) and fighter-interceptors (FI).

Even though these two forms of defenses cannot be separated geographically like the FAD and BSAM, this model makes the assumption that they can be treated independently of each other. This means that an individual bomber's probability of being killed by the RASAM system is independent of his probability of being shot down by the FIs. The remainder of this section is addressed to RASAMs, while FIs are dealt with in the next section.

A RASAM defense differs in two major ways from the BSAMS. First, the depth of the coverage is no longer a neglible factor. It is

assumed that the RASAMs may be encountered anywhere within the defended territory. Instead of a barrier which a bomber must penetrate once and can then ignore for the remainder of the mission, RASAMs present a continuous threat from the time penetration begins until the bomber leaves the defended area.

The second difference is related to the first. Since RASAMs are distributed throughout the defended area, they are necessarily interspersed with the targets in the area. A bomber need not penetrate through the entire depth of the RASAM net to reach his targets. Some targets may be reached only a short way into the net; more targets will be exposed as deeper penetration is made.

Although the above difference between RASAMs and BSAMs are important in the model's treatment of the former, there are also important similarities between the two. Since bombers could be expected to fly around or suppress any known SAM sites, the assumption is made again that, because of their location uncertainty, the only SAMs of interest to the model are mobile SAMs. Also, it is assumed that the RASAMs are dispersed randomly and uniformly through the defended area. In practice, the placement of the RASAMs might be affected by the defense's knowledge of the locations of the targets within the defended area. However, if the offense believed that the actual distribution of RASAMs was significantly different from uniform they might find it advantageous to plot courses around suspected high concentrations of RASAMs.

The RASAM defense is shown in Figure 3.7.

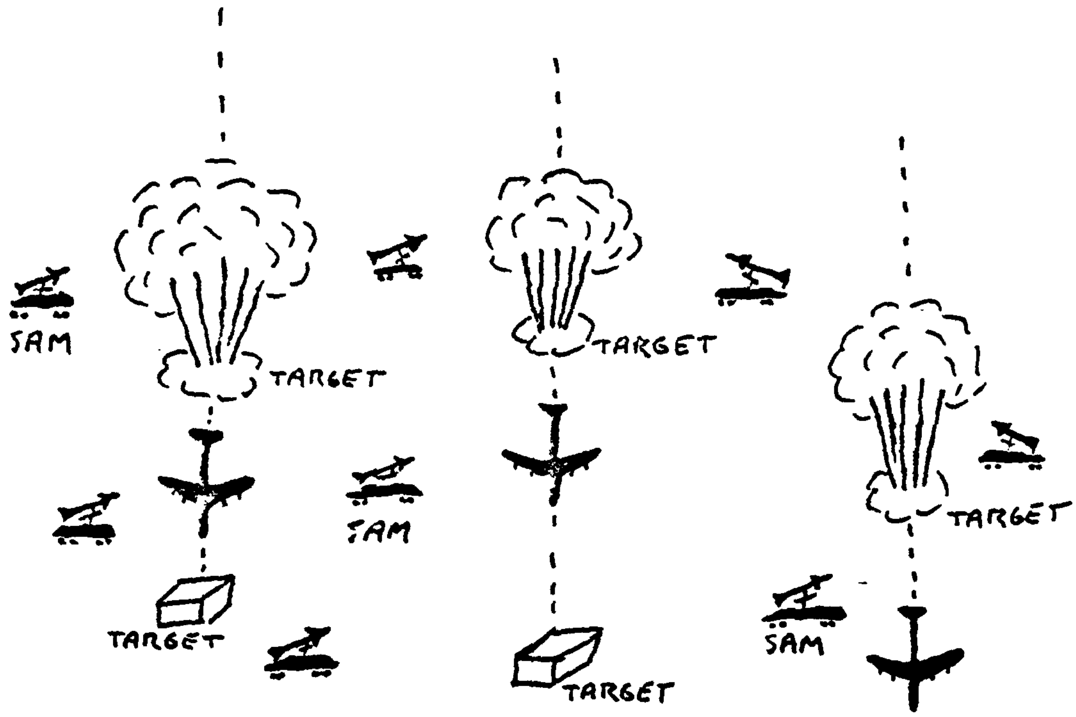

FIGURE 3.7

Random Area SAMs.

FIGHTER-INTERCEPTORS

The second form of area defense that bombers may encounter is fighter-interceptors (FI). As a bomber penetrates into the defended area, it will come within range of fighters stationed at various airbases throughout the country. As the bomber slips within range of a base, any fighters stationed at that base which are not already engaged with other bombers may be assigned to attack it. As in the case of the FAD, these fighters may be either raid controlled or close controlled.

In the course of the penetration, there may be many such FI attacks on the bomber. The number will depend on the bomber's total

depth of penetration, the number of airbases within FI range of the bomber's course, and the FI commitment policy of the defense. It may also depend on the total number of offensive penetrators since this will be one factor determining the number of FIs available to attack a particular bomber.

In the case of close controlled intercepts, a typical bomber-interceptor engagement will follow roughly the same scenario presented in the FAD section. The fighter would be vectored by the control system to an appropriate intercept point. There it would search for the bomber and after detecting it would attempt to convert on it. The bomber can to some extent degrade the FI's capability to convert on it by jamming the FI's radar, denying the FI range information. If the FI successfully converts on the bomber, it will then release one or more air-to-air missiles. After the engagement, the fighters' remaining fuel and armament loads and the location of other detected bombers will determine whether it can attack a second bomber or must return to base to be recycled.

To further define the specific problem being addressed, several assumptions are made by this model.

The first assumption concerns the FI force. There are over a dozen different fighters and fighter-interceptors in the Soviet inventory. They range in capabilities from the subsonic MiG-17, dating back to the early 1950's and having a range of under 300 nm, to the all-weather SU-15, with a nearly 400 nm radius and capable of speeds in excess of Mach 2, and the MiG-25 with a 600 nm radius and capable of speeds of Mach 2.8 (Ref 10; 14:178-222). As was done for the SAMs, this model assumes a homogeneous FI force having capabilities that are

average or typical of the various fighter-interceptors that bombers could actually expect to encounter. These capabilities need not necessarily be the same as those of the FAD fighter force.

The next assumptions deal with the defensive command and control system. It is assumed in this model that all the fighter-interceptors are close controlled, as was the case in the FAD defense. Unlike the FAD system, it is assumed that in the area defense the interceptors are controlled from a ground based detection and control system rather than SUAWACS. This GCI net will detect approaching bombers and then vector available FIs to appropriate intercept points. The scenario is shown in Figure 3.8. The model assumes that the chance of an encounter between a previously undetected bomber and an airborne FI on CAP or enroute to an assigned intercept point in neglible. The model also assumes that the GCI net and the RASAMs operate independently.

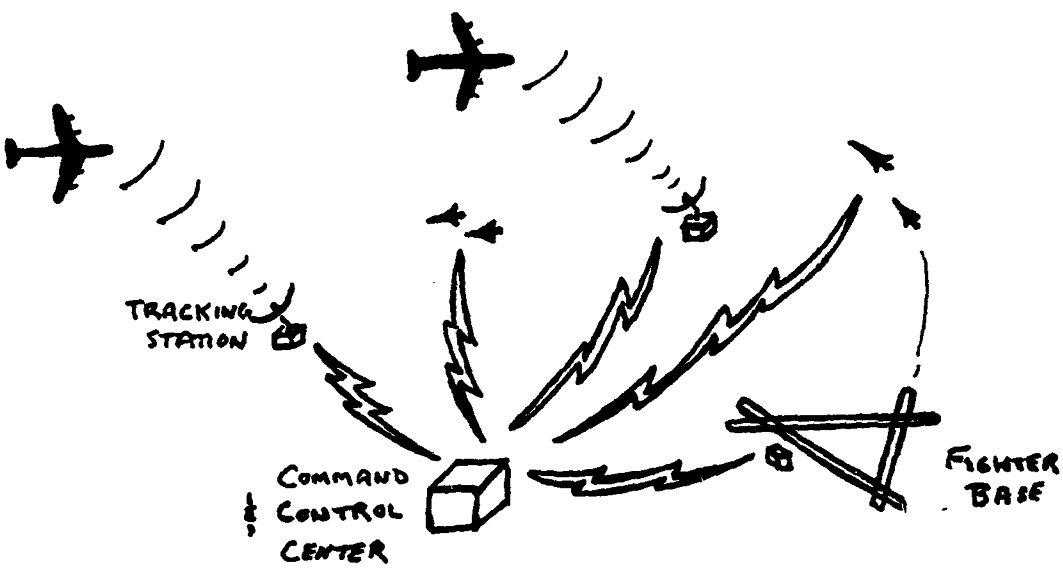

FIGURE 3.8

Area fighter-interceptor defense.

To effectively close control the interceptors, the defense must be capable of tracking bombers over the entire defended area. It is assumed therefore that the defensive GCI net is spread over the defended area. As the bomber begins penetration, he may be able to avoid detection by the net for some time by flying at low altitudes. However, once the bomber is detected by the radar net, it is assumed that the defense will be able to track the bomber's course and vector interceptors to it for the remainder of the mission.

Several other assumptions will be introduced as the FI part of the model is developed. They will be described in the section on FI area defenses in Chapter V.

TERMINAL SAM, WEAPONS DELIVERED, AND VALUE EXTRACTED

The assumptions and analysis involved in any model's representation of terminal SAM defenses, weapons delivered, and value extracted or destroyed by the offense are very interrelated. These three areas are treated together in this section.

It seems reasonable to assume that the offense and defense will be in rough agreement on what makes up the target set. Differences may arise, however, in the relative values each places on the targets within the target set. The defense will try to defend the targets in a way it finds optimal according to the relative values it puts on the targets. The offense will try to optimize its attack using its own value system, which may differ from that of the defense.

It has been shown that the optimum strategy for the defense is to defend each target relative to the other targets in proportion to that target's value relative to the other targets (Ref 13:9). In other

words, if on some scale of value, the defense considers target B to be twice as valuable as target A, then they should defend target B with twice as many SAMs as are used at target A. This system assumes that it is possible to make such relative value judgments and also assumes that two SAMs can defend a target twice as effectively as one. If these assumptions are accepted, then for a given number of defensive SAMs and a given target set of known relative values the optimal defense can be found.

If the offense places the same relative values on the targets as the defense does, then the above system will present the offense with a constant price that must be paid to extract each unit of value. Using the example above, the offense must expend twice as many resources to be sure of destroying target B as it would to destroy target A. This would leave the offense indifferent to which target it attacked since the total value it could destroy would be proportional to its total resources no matter which system it used to select targets to attack.

The situation is different if the offense places different relative values on the targes than the defense. In this case, the offense can optimize its attack by attacking those targets which it perceives as being defended to a level less than their value compared to other targets. Still using the above example, if the offense perceives targets A and B as equally valuable then it will choose to attack target A since it is defended only half as much as target B.

So far, the entire analysis presented above has been based solely on considerations involving relative values of various targets.

Another important factor that has thus far been ignored is a target's distance inside the defended area. Consider again two targets A and B of equal value, but now let target B lie at a greater depth inside the defended area than target A. The offense will be more likely to attack target A since a bomber will stand a greater change of penetrating to A through the area defenses than it would of penetrating the longer distance to target B. Knowing this, the defense may increase the level of terminal defenses at target A to try to make the offense indifferent to the two targets.

To make even a reasonably detailed probabilistic model of the situation described above would require several things. Some knowledge of the target set and the relative values of the targets would, of course, be necessary. Then it is possible that one would require some form of weapon allocation scheme to determine the number of weapons to assign to each target. Finally, a distribution would have to be found giving the probability that a target is destroyed as a function of the level of defense and the number of offensive penetrators assigned to attack it.

All that is beyond the scope of this model. To simplify the problem into something that can be more easily modelled, the following two assumptions are made. First, it is assumed that all targets are of equal value and are perceived to be so by both offense and defense. Second, it is assumed that neither side considers the location of the target when planning its attack or defense. These assumptions clearly differ significantly from reality but will allow some representation of the terminal defense and value extracted aspects of the bomber mission without making the model unreasonably complicated.

The above assumptions lead directly to three others. First, if all the targets are of equal value, then it can reasonably be assumed that they will all be uniformly defended. In other words, each target will be defended by the same number of terminal SAMs. Second, it can also be assumed that the offense will allocate the same number of weapons to each target. Finally, if each target is of equal value, then it follows that the total value destroyed by the offense will be directly proportional to the number of targets destroyed.

To determine the number of targets destroyed, two more assumptions will be made. It will be assumed that only one weapon is released at each target and that weapon will totally destroy the target if it detonates where it is aimed. In other words, the weapon either succeeds, in which case the target is destroyed, or fails, in which case the target is unharmed. There is no partial damage to the target.

RECOVERY

After a bomber has released its last weapon, the final part of its mission will be to get out of the defended country and find a safe place to land. If the bomber succeeds in doing that, then a variety of factors at the time will determine if it can ever be refueled, rearmed, and sent on a second mission.

A complete analysis of all parts of the recovery problem is beyond the scope of this model. This model will only deal with the bomber's ability to get out of the defended area. All subsequent aspects of the problem such as locating a safe base at which to land will not be considered.

This model assumes that after a bomber has released its last weapon it will continue to penetrate south along the corridor until it exits at the bottom, or southern end. At this point tne bomber will be considered to be outside the defended area and its mission, in terms of this model, ended.

It seems reasonable to assume that the defense will not know when the bomber has exhausted its weapons and is no longer a threat. Therefore, attacks on the bomber, both by fighter-interceptors and SAMs, will continue until the bomber leaves the defended area. In other words, the bomber continues to face similar defenses while leaving the defended area as he did when penetrating to reach his targets. The only difference is that there is assumed to be no BSAM defense at the far south southern end of the corridor. The defense is not concerned about an attack from the south and is not interested in destroying bombers that are leaving the defended area and are clearly no longer a threat.

CRUISE MISSILES

Up until now, the subject of cruise missiles and their effect on the mission of the air breathing element has not been addressed. This will be the subject of this section.

To begin with, several assumptions will be made. First, it will be assumed that each cruise missile is released at its maximum range from its target. Compared to other weapons, cruise missiles are fairly heavy. The farther they are carried by the bomber, the more fuel is consumed by the bomber which in turn limits the amount of other ordnance the bomber can carry. Furthermore, each individual penetrator may have a better chance or survival the more penetrators there are.

If a bomber gets shot down, the total loss to the offense will be less if he is not still carrying his cruise missiles. These are two reasons for the bombers to release their cruise missiles as early as possible so the above assumption seems reasonable.

Consistent with the discussion of value destroyed in the previous section, the next assumption is that only one cruise missile is fired at each target and if the cruise missile detonates at its target, the target will be completely destroyed. The targets of the cruise missiles will not be the same as those of the bombers themselves.

The last general assumption is that all the defenses that threaten the bombers may also threaten the cruise missiles. The various defenses may not be equally effective against cruise missiles as against bombers but they have the potential to be a threat. This means that cruise missiles will face TSAMs, fighter-interceptors, RASAMs, and if they are released early enough, BSAMs and even FAD interceptors.

The forward air defense part of the model would become significantly more complicated if cruise missiles were released during the FAD engagement and so, for modelling convenience, it will be assumed that this does not occur. That is, the cruise missiles may be released before or after the FAD, but not during the actual engagement. Also, in this model, the depth dimension of the BSAM defense has been assumed to be insignificant, so it is reasonable to assume that cruise missiles will not be released within the BSAM band. Thus, within the structure of this model, there are three different possible assumptions that can be made about cruise missile release points.

The first assumption would be that all the cruise missiles are of fairly short range and are going to targets reasonably deep within the

defended area. In this case, the bombers would not release the cruise missiles until after they had passed through the BSAM defense and begun actual penetration of the defended area.

A second case would involve cruise missiles of longer range. In this case it might be reasonable to assume that the bombers release their cruise missiles after penetrating through the FAD zone but before entering the BSAM band and beginning actual penetration. In practice, the FAD zone might extend right up to the coastline of the continent and the BSAM band which would leave no place between the two layers of defense for the bombers to release their cruise missiles. Still, the above assumption might be considered equivalent to having the bombers release the cruise missiles so late in the FAD engagement that there is no significant effect on the outcome of the engagement.

The third possible case would involve cruise missiles of extremely long range in which case it could be assumed that the cruise missiles are all released before the bombers enter the FAD zone. At this time, that may not be a realistic assumption except possibly in cases where the cruise missiles are all destined for targets in the far northern part of the defended area. Nonetheless, it is included here for completeness.

The cruise missiles currently being produced by Boeing have a range of over 1500 miles. Thus, in an actual mission, the most likely case would be some combination of the above involving at least the first two and possibly all three cases. In other words, for cruise missiles of a given range and a target set distributed over the entire defended area, some of the cruise missiles may be released before the bombers encounter the BSAM defense or even before the FAD zone while others,

those going to southern targets, will have to be carried through the BSAM band and partway through the defended area. In this model, the three cases are treated separately; however, with a few minor additional assumptions, it should not be difficult to represent combinations of cases.

CHAPTER IV

PRIOR TO PENETRATION

Purely for convenience, the various parts of this model have been divided up into three groups which will be described in this chapter and the two chapters following. This chapter covers the parts of the model that deal with the stages of the bomber's mission that take place before actual penetration of the defended area begins. In this model, actual penetration is somewhat arbitrarily defined to begin after the bomber has gone through the BSAM defense. Therefore, the parts of the model described in this chapter are base escape (BE), airborne refueling (AR), forward air defense (FAD) and the barrier SAMs (BSAMs).

BASE ESCAPE

Consider a base at which there are stationed B bombers. In a preemptive attack by SLBM foreces, a total of M warheads are targeted at the base. Assume temporarily that at the time of the attack, all the bombers are on the ground. The problem is to find a distribution for the number of bombers that survive the attack.

When warning is first received of the attack, the bombers will begin to take off and fly away from the base. It is assumed that when the warheads arrive and detonate, all bombers that are not airborne are destroyed. Bombers that are airborne will have some probability of survival but will not be assured of surviving. The problem can now be separated into two parts; first finding the number of bombers airborne at the time of warhead detonation and second, finding the probability that an airborne bomber will survive the attack.

To find A, the number of bombers airborne when the missiles detonate, it is necessary to know the amount of warning the base has received of the attack and the rate at which bombers can take off from the base. The takeoff rate is assumed to be a constant depending on the number of runways at the base and the interval between takeoffs of consecutive bombers. The warning time, T_W, is a random variable that could depend on the total missile flight time, the time at which the missile is detected after breaking water, and the command and control time required to evaluate the data, make a decision and notify the base of attack.

When warning of the attack is first received, there may be a delay before bombers can start taking off due to the reaction time of the air-crews. Call this delay T_R. The total amount of time available for the bombers to take off in is then T_A, where

$$T_A = T_W - T_R \qquad 4.1$$

The maximum theoretical time in which bombers could take off is the total of missile flight time, T_M, while the minimum is zero. The former would require immediate detection of the missiles as they break the water and crews already aboard the bombers; the latter would occur if the warning time, T_W, was less than the reaction time, T_R. The most likely time would be somewhere between the two extremes. Assume that the probability density function of the time, $p_{T_A}(t)$, can be fully specified. It might have a shape similar to the one in Figure 4.1.

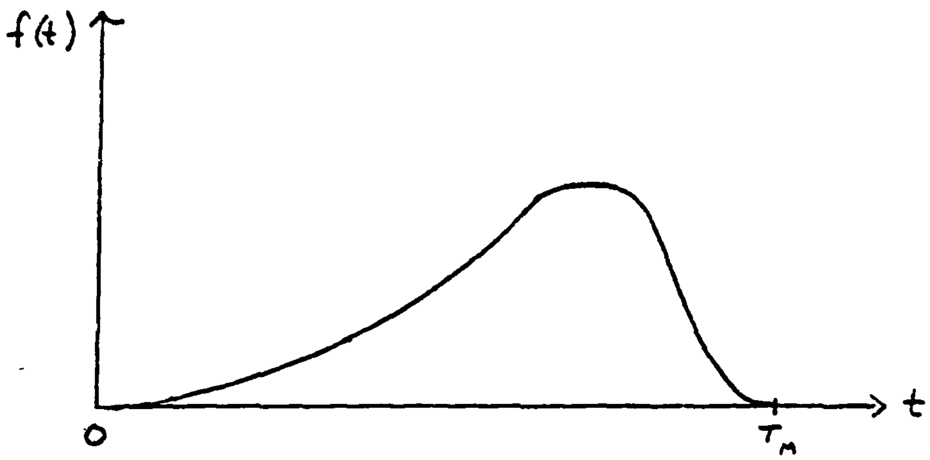

FIGURE 4.1

Probability density function for T_A

With the assumption of a constant take off rate, the number of bombers airborne when the warheads detonate is given by

$$A = \min (\langle rT_A \rangle, B) \tag{4.2}$$

where r is the rate at which bombers take off. The pointed brackets are read "greatest integer in" and account for the fact that the number of bombers airborne must be an integer.

The probability that an airborne bomber is killed can be found by taking the ratio of the volume of airspace around the detonating missiles within which the bomber would be destroyed to the total volume of airspace in which the bomber could be located.

It is assumed that a detonating warhead has some lethal radius, r_L, within which any bomber is destroyed and outside of which a bomber

will survive undamaged. When the warhead detonates, there will be a lethal volume, V_ℓ, around it inside of which any bomber is killed with probability one. If the warhead is assumed to be detonated above the ground, then this volume can be approximated by a sphere.

$$V_\ell = (4/3)\pi r_L^2 \tag{4.3}$$

Bombers outside this volume survive with probability one. For the case of an attack involving M warheads, an upper bound for the total lethal volume, V_L, of airspace is

$$V_L = (4/3)M\pi r_L^3 \tag{4.4}$$

This is the case where the lethal volumes from individual warheads do not overlap. It is the most offense conservative case and to avoid the problem of trying to calculate the overlaps, this is the case employed in the model.

The total volume within which bombers may be located can be described by a cylinder centered on the base with a radius R and height H given by

$$\begin{aligned} R &= v_h T_A \\ H &= v_c T_A \end{aligned} \tag{4.5}$$

where v_h and v_c are the horizontal velocity and rate of climb of a bomber. The total volume of the cylinder is

$$\begin{aligned} V_T &= \pi R^2 H \\ &= \pi v_h^2 v_c \, T_A^3 \end{aligned} \tag{4.6}$$

If the assumption is made that airborne bombers are equally likely to be anywhere inside V_T then the probability that an airborne bomber is killed, given T_A is the ratio of the two volumes.

$$P_{k|T_A} = \max\left(1, \frac{4Mr_L^3}{3v_h^2 v_c T_A^3}\right) \qquad 4.7$$

The function has been written to indicate that the probability of being killed cannot be greater than one.

From Equation 4.7, the probability that an airborne bomber survives the attack given T_A is

$$\begin{aligned} P_{S|\cdot A} &= 1 - P_{K|T_A} \\ &= 1 - \max\left(1, \frac{4Mr_L^3}{3v_h^2 v_c T_A^3}\right) \end{aligned} \qquad 4.8$$

It is assumed that for a given T_A, each airborne bomber's survival event is dependent of the survival events of the other bombers. The distribution of the number of bombers that survive for a given T_A, is therefore binomial

$$\begin{aligned} P'_{be}(K|T_a) &= \binom{A}{k} P_{S|t}^{\ k} (1-P_{S|t})^{A-k} \\ &= \binom{\min(<rt>, B)}{k} \left[1-\min\left(1, \frac{4Mr_L^3}{3r_n^2 r_c T_A^3}\right)\right]^k \\ &\quad \left[\min\left(1, \frac{4Mr_L^3}{3v_n^2 v_c T_A^3}\right)\right]^{\min(<rt>, B) - k} \end{aligned} \qquad 4.9$$

The distribution of the number of bombers that survive can now be found by conditioning on T_A.

$$P'_{be}(k) = \int_{k/r}^{T_M} \binom{\min(<rt>, B)}{k} \left[1 - \min\left(1, \frac{4Mr_L^{\,3}}{3v_h^{\,2}v_c t^3}\right)\right]^k$$

$$\left[\min\left(1, \frac{4Mr_L^{\,3}}{3v_h^{\,2}v_c t^3}\right)\right]^{\min(<rt>, B) - k} \qquad 4.10$$

In order for k bombers to survive, at least k must be airborne at the time the missiles detonate. From Equation 4.2, it is found that the smallest value of T_A which would allow k bombers to be airborne is k/r. This explains the lower limit on the integration.

It would be possible to numerically integrate equation 4.10 on a computer to obtain the exact distribution of the number of bombers that survive the attack. However, it may be more convenient to use the following approximation.

First break the integral up as follows:

$$P'_{be}(k) = \int_{k/r}^{(k+1)/r} \binom{\min(<rt>,b)}{k} P_{S|t}^{\,k}(1-P_{S|t})^{\min(<rt>,B)-k} p_{T_A}(t)dt$$

$$+ \int_{(k+1)/r}^{(k+2)/r} \binom{\min(<rt>,B)}{k} P_{S|t}^{\,k}(1-P_{S|t})^{\min(<rt>,b)-k} p_{T_A}(t)dt$$

$$+ \;.\;.\;.$$

$$+ \int_{(B-1)/r}^{B/r} \binom{\min(<rt>,B)}{k} P_{S|t}^{k}(1-P_{S|t})^{\min(<rt>,B)-k} p_{T_A}(t)dt$$

$$+ \int_{B/r}^{T_m} \binom{\min(<rt>,B)}{k} P_{S|t}^{k}(1-P_{S|t})^{\min(<rt>,B)-k} p_{T_A}(t)dt \qquad 4.11$$

Consider the value of <rt> in each of the integrals in Equation 3.11.

$$<rt> = k+i \quad \forall t:(k+i)/r \leq t <(k+i+1)/r \qquad 4.12$$

This implies that max(<rt>), B) will be constant in each interval and can be taken outside the integrals.

If one assumes that $P_{S|t}$ is nearly constant within each interval, then it can be approximated by $P_{S|t_i}$ where t_i is the average value of t within the i^{th} interval. If this approximation is made, then $P_{S|t_i}^{k}(1-P_{S|t_i})^{\min(<rt>,B)-k}$ can also be taken outside each integral, giving

$$
\begin{aligned}
P'_{be}(k) = &\binom{k}{k} P_{S|t_1}^{k}(1-P_{S|t_1})^{k} \int_{k/r}^{(k+1)/r} p_{T_A}(t)dt \\
&+ \binom{k+1}{k} P_{S|t_2}^{k}(1-P_{S|t_2})^{k+1-k} \int_{(k+1)/r}^{(k+2)/r} p_{T_A}(t)dt \\
&+ \ldots \\
&+ \binom{B-1}{k} P_{S|t_{B-k}}^{k}(1-P_{S|t_{B-k}})^{B-1-k} \int_{B-1/r}^{B/r} p_{T_A}(t)dt \\
&+ \binom{B}{k} P_{S|t_{B-k+1}}^{k}(1-P_{S|t_{B-k+1}})^{B-k} \int_{B/r}^{T_m} p_{T_A}(t)dt
\end{aligned}
\qquad 4.13
$$

Now consider the integrals remaining in equation 4.13. Each integral can be interpreted as the probability that T lies between the upper and lower bounds of integration. However, by Equation 4.2, this probability can further be interpreted as the probability that the number of bombers airborne when the warheads detonate is r times the lower limit of integration. Matematically,

$$
\begin{aligned}
\int_{(k+i)/r}^{(k+i+1)/r} p_{T_A}(t)dt &= P[(k+i)/r \le T_A < (k+i+1)/r) \\
&= P(k+i \le rT_A < k+i+1) \\
&= P(k+i \le <rT_A> < k+i+1)
\end{aligned}
$$

$$= P[\max(k+i,B) \leq \max(<rT_A>,B) < \max(k+i+1,B)]$$

$$= P(A = \max(B,k+i)) \qquad 4.14$$

Substituting equation 4.14 into 4.13 and simplifying (substitute j+k+i), one finds

$$P'_{be}(k) = \sum_{j=k}^{B} \binom{j}{k} P_{S|t_i}{}^{k} (1-P_{S|t_i})^{j-k} P(A=j) \qquad 4.15$$

In deriving equation 4.15, the assumption was initially made that all the bombers at the base are on the ground when the attack occurs. In reality, it is quite probable that some of the bombers would be on airborne alert at the time of the attack. These bombers can be assumed to survive the attack on the base with probability one. If B_A is the number of bombers airborne and B_G is the number on the ground at the time of attack where B_A and B_G sum to B; then with this assumption, the distribution of the number of bombers that escape can be written

$$\begin{aligned} P_{be}(k) &= 0 & k < B_A \\ &= P'_{be}(k-B_A) & B_A \leq k \leq B \\ &= 0 & k \geq B \end{aligned} \qquad 4.16$$

The above expressions apply to a single base. If one considers many bases having different characteristics such as different numbers of bombers assigned to them or different distances to the point of SLBM launch, then to get the total number of bombers that escape, one must sum over all the bases. For example, for n bases:

$$P_{BE}(k) = \sum_{k_1}^{B_1} \sum_{k_2}^{B_2} \cdots \sum_{k_{n-1}}^{B_{n-1}} P_{be}^{(1)}(k_1) P_{be}^{(2)}(k_2) \cdots\cdots P_{be}^{(n-1)}(k_{n-1})$$

$$P_{be}^{(n)}(k-k_1-k_2-\cdots\cdots\cdots - k_{n-1}) \quad 4.17$$

Alternatively, if one assumes the bases are uniform in every respect, including having the same number of warheads assigned to each one, then the entire force can be treated as one large base having nB_A bombers in the air, nB_G bombers on the ground, and a bomber takeoff rate of nr. T_M, the time of flight of the missiles, will remain the same.

It may be that in some cases the assumption of identical bases would differ more from reality than the modeller wished to allow, in which case Equation 4.17 would have to be used. However, because of the savings in computational difficulty to be gained, this model assumes that all bases are identical and treats the bomber force as one large base.

REFUELING

The next problem is that of aerial refueling. Consider a force of B_{BE} bombers that have successfully escaped the SLBM attack on their bases and now desire inflight refueling. These bombers are to be refueled by a force of T tankers. Some of the tankers, T_S, either because they are airborne at the time of attack or because they are stationed at a "safe" base, are assured of surviving the initial SLBM attack. The remaining tankers, T_G, are on the ground at bases that are attacked and so face a base escape problem identical to that of the bombers. Given these two forces of aircraft, the problem is to

find a distribution for the number of bombers that are successfully refueled.

Consider first the case where B_{BE} is less than or equal to T_S. In this case there will likely to be enough tankers to refuel all the bombers, and the number of bombers that can get refueled is limited only by the total number of bombers, B_{BE}, requiring refueling. Each bomber is assumed to have some probability P_{AR} of successfully refueling given an available tanker with sufficient fuel. Since it is assumed that each bomber's refueling event is independent of all the other bombers' refueling events, the distribution of bombers that successfully refuel is binomial.

$$P_{AR}(k|B_{BE} \leq T_S) = \binom{B_{BE}}{k} P_{AR}^{k}(1-P_{AR})^{B_{BE}-k} \qquad 4.18$$

Consider now the case where B_{BE} is greater than T_S. In this case it is possible, although not necessarily inevitable, that there will be more bombers needing refueling than tankers available to service them.

If enough of the T_G tankers facing the base escape problem survive to bring the total number of airborne tankers up to at least B_{BE}, then the probability that k bombers get refueled will be given by equation 4.18 above. For this to occur, the number of tankers, i, that survive the base escape problem must be greater than or equal to the difference between B_{BE} and T_S. The probability of this is

$$P(i \geq B_{BE} - T_S) = \sum_{i-B_{AR}-T_S}^{T_G} P_{TE}(i) \qquad 4.19$$

$P_{TE}(i)$ is the probability that i tankers of the T_G on the ground survive the base escape. The functional form is the same as $P_{BE}(i)$ for bombers given in Equation 4.16; however, the parameters such as velocities, take off rate, and lethal radius with respect to a warhead need not be the same as for bombers.

If, of the T_G tankers, some number j less than the difference between B_{BE} and T_S survive then the number of bombers that can be refueled will be limited by the number of tankers available rather than the number of bombers. In this case the probability that k bombers get refueled is

$$
\begin{aligned}
P_{AR}(k|B_{BE} > T_S+j, j) &= \binom{T_S+j}{k} P_{AR}^{k}(1-P_{AR})^{T_S+j-k} \qquad k > T_S+j \\
&= 0 \qquad \text{ow.}
\end{aligned}
\tag{4.20}
$$

Using equations 4.18, 4.19, and 4.20, and conditioning on the number of tankers that survive, one can now find the distribution of the number of bombers that get refueled for the case where B_{BE} is greater than T_S.

$$
\begin{aligned}
P_{AR}(k|B_{BE}>T_S) = {} & \binom{B_{BE}}{k} P_{AR}^{k}(1=P_{AR})^{B_{BE}-k} \sum_{j-B_{BE}-T_S}^{T_G} P_{TE}(j) \\
& + \sum_{j=\max(0,k-T_S)}^{B_{BE}-T_S-1} \binom{T_S+j}{k} P_{AR}^{k})^{T_S+j-k} P_{TE}(j)
\end{aligned}
\tag{4.21}
$$

FORWARD AIR DEFENSE

Next consider the problem of forward air defense. Temporarily set aside the question of bombers penetrating at high altitudes versus penetrating at low altitudes and just look at a homogeneous force of

force of B bombers penetrating through a forward air defense zone defended by a force of F homogeneous fighters.

It is assumed that, with SUAWACS, the defense can tell exactly or make a good estimate of the total number of bombers in the force. To maximize their effectiveness, measured in expected number of bombers killed, the defense will attempt to assign a uniform number of fighters to each bomber as it enters the FAD zone. If F is an integer multiple of B,

$$F = iB \quad , \tag{4.22}$$

then there will be i fighters assigned to each bomber. Otherwise,

$$F = iB + R \tag{4.23}$$

where R is some remaining number of fighters less than B. In this case, there will be i+1 fighters assigned to the first R bombers to enter the FAD zone and i fighters assigned to the other B-R bombers.

Now consider one of the bombers and the n fighters assigned to it. Each fighter will have some probability of detecting and converting on the assigned bomber, P_{dc}, and another probability, $P_{k|dc}$, of killing the bomber after detection and conversion. Together, these two probabilities will give the probability P_k that an assigned fighter kills the bomber

$$P_k = P_{k|dc} P_{dc} \tag{4.24}$$

Assume the successive attacks by different fighters on the same bomber are independent. This might be because the various fighters arrive in the vicinity and detect the bomber at different times or

simply because the fighter tactics against bombers do not improve an individual fighter's probability of killing the bomber when attacking in pairs or groups over the probability when attacking by turn. If this is the case then in order for the bomber to survive the FAD, it must survive the attacks of all n fighters assigned to it. Since homogeneous fighters have been assumed, each will have the same P_k. Therefore, the bomber's probability of survival, P_s, can be expressed

$$P_s = (1 - P_k)^n \qquad 4.25$$

It is assumed that each fighter can only attack the bomber it was assigned to. Thus, there is no link between the killing of one bomber and the survival of another since the fighters that have just made the kill cannot go to assist fighters attacking the other bomber. Each bomber's survival is therefore independent of what has happened to the other bombers after the initial interceptor assignment. If all the bombers have the same probability of surviving, i.e., each is attacked by the same number n of fighters, then the distribution of the number of bombers that survive is binomial:

$$P(k \text{ bombers survive} \mid B) = \binom{B}{k} P_s^{\,k}(1-P_s)^{B-k} \qquad 4.26$$

It has already been noted that if F/B is non-integer, there will be two distinct groups of bombers, a group of R bombers having <F/B>+1 fighters assigned to attack each one and a group of B-R bombers with <F/B> fighters assigned to each. Within each group, the probability of survival for each bomber is the same:

$$P_{S1} = (1-P_k)^{<F/B>+1} \qquad 4.27a$$

and

$$P_{S2} = (1-P_k)^{<F/B>} \qquad 4.27b$$

The distribution of survivors within each group is binomial and the combined distribution for the total number of bombers that survive from both groups can be found by conditioning on the number that survive from one group.

$$\begin{aligned} P(k \text{ survive FAD}|B) &= \sum_i P(k \text{ surv}|i \text{ of } R \text{ survive})P(i \text{ of } R \text{ surv}) \\ &= \sum_i P(k-i \text{ of } B-R \text{ surv})P(i \text{ of } R \text{ surv}) \\ &= \sum_{i=\max(0,k=B+R)}^{\max(k,R)} \binom{B-R}{k-i} P_{S2}^{k-i}(1-P_{S2})^{B-R-k+i} \\ &\qquad \binom{R}{i} P_{S1}^{i}(1-P_{S1})^{R-i} \end{aligned} \qquad 4.28$$

This completes the basic analysis of the Forward Air Defense problem. The only question that remains is how to deal with two different groups of bombers, those flying at high altitude and those flying at low altitude.

This model is interested in the distribution of bombers from each group that survive the forward air defense, as opposed to a distribution of the total number of bombers from both groups that survive. Thus, the problem becomes one of deciding how to assign fighters to the two groups. The probabilities a fighter detects, converts on and kills a bomber may be smaller at low altitudes than at high altitudes but they will be the same for all fighters attacking low flying bombers and the same for all fighters attacking high flying bombers. All the independence assumptions already discussed still apply. Knowing the

number of bombers at each altitude and the number of fighters assigned to each group of bombers, one can apply the above analysis to each group to find the desired distributions of the numbers of high and low altitude bombers surviving the FAD.

It is still assumed that the defense assigns fighters uniformly to all the bombers. Hence, if the number of fighters is an integer multiple of the number of bombers, then each bomber has F/B fighters assigned to it, regardless of whether it is flying at high or low altitude.

The only question that arises is how to apportion the R remaining fighters if the ratio of fighters to bombers is not an integer. For the purposes of this model, it is assumed that the remaining fighters are assigned, one each, first to the high altitude bombers and then, if any fighters still remain, to the low altitude bombers. This is a somewhat arbitrary assignment but as justification consider that bombers flying at high altitude will most likely be detected earlier than those flying at low altitudes. If the defense assigns the extra fighters to the first R bombers that are detected, then these fighters will probably be assigned to bombers at high altitude.

BSAM

After passing through the forward air defense zone, the bombers will reach the continent. The last line of defense they will encounter before beginning penetration is the barrier SAMs.

Before approach the problem of the barrier of SAMs as a whole, consider for the moment a single SAM site and a bomber penetrating through the site's coverage. The problem can be mathematically

described using two variables. Define x to be the lateral offset distance between the path of the bomber and the center of the SAM site (the distance between the bomber and SAM site at the bomber's point of closest approach). Let $P_k(x)$ be the probability that a bomber flying at a specified altitude is killed by the SAM site if the lateral offset distance is x. A sketch of the function might look something like that shown in Figure 4.1, where x_{max} is the maximum range of the SAM.

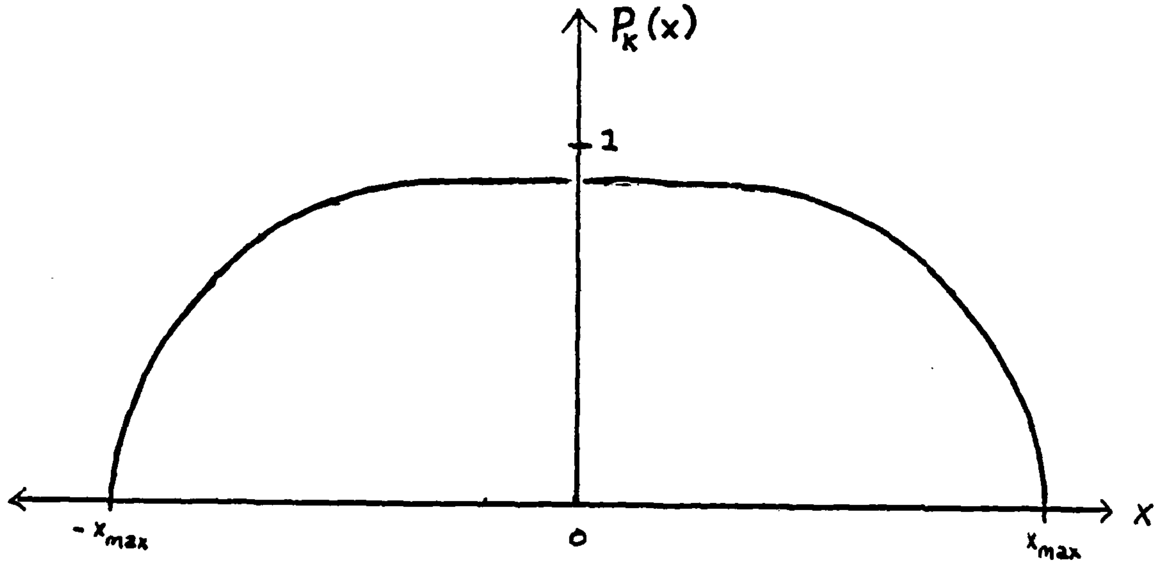

FIGURE 4.2

SAM probability of kill on bomber vs offset distance.

For the purposes of this model, the curve in Figure 4.1 is reduced to two parameters, a probable lethal diameter (PLD) of the SAM site and a constant probability of kill, P_{ks}, within the PLD. The probability of kill outside the PLD is assumed to be zero. (See Figure 4.3) Since

the effectiveness of the SAM may vary with the altitude of the target, the PLD and P_{ks} may be different when considering bombers at two different altitudes. Both the PLD and P_{ks} will be inputs to the model, one set for each altitude.

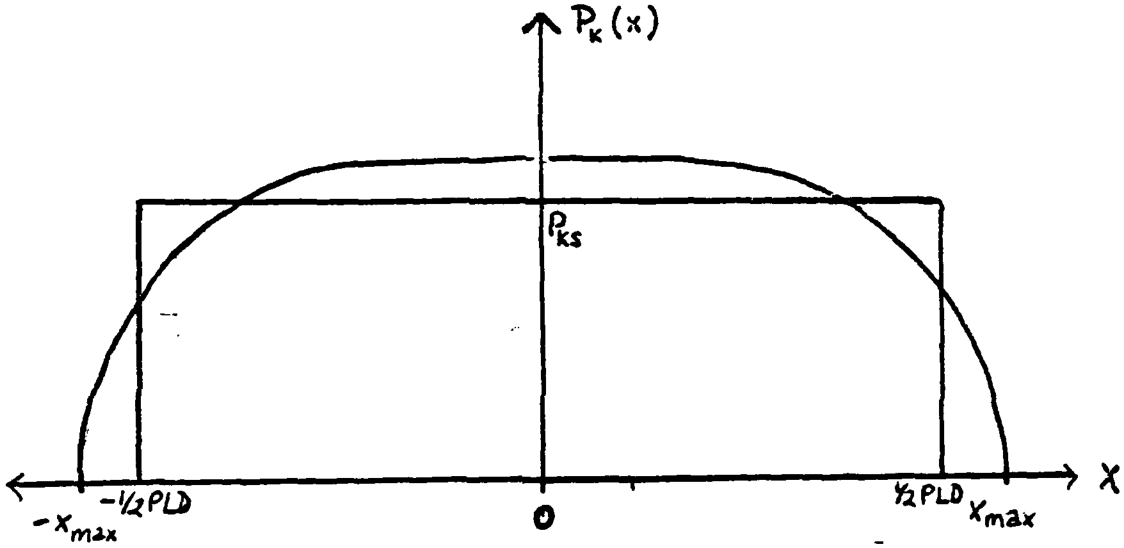

FIGURE 4.3

PLD and P_{ks} of SAM site.

Now consider the problem of a barrier of N_B SAM sites, all having the same PLD and P_{ks}. The sites are distributed uniformly along the width of the band, W.

To find the probability that a bomber survives the barrier SAMs, look first at the case where the bomber approaches a barrier with only one site. The site has a width of coverage defined by its PLD and since it can be located anywhere along the barrier, the bomber's

probability of crossing through the site's coverage is the ratio of the site's PLD to the total width of the barrier. If the bomber crosses the site's coverage, the probability that it is killed is P_{ks}. The probability that the bomber survives is therefore

$$P_{ss} = 1-(w/W)P_{ks} \qquad 4.29$$

where w is the PLD of the SAM site.

Now if there are N_B identical SAM sites in the barrier and the location of each and its P_{ks} are independent of all the other sites, then the event that the bomber survives any site is independent of its event of surviving any other site. The probability that the bomber safely penetrates through the barrier is the probability that it survived all N_B sites which is

$$P_{BS} = [1-(w/W)P_{ks}]^{N_B} \quad . \qquad 4.30$$

An alternative way of deriving the above result is to condition the bomber's probability of survival on the number of SAM sites it encounters. The probability that the bomber encounters an individual SAM site is

$$P = w/W \quad . \qquad 4.31$$

If there are N_B sites in the barrier, each located independently of the others, then the distribution of the number of sites encountered by the bomber is binomial

$$P(\text{encounter } k \text{ sites}) = \binom{N_B}{k} p^k (1-p)^{N_B-k} \quad . \qquad 4.32$$

The bomber's probability of survival if it encounters k sites is

$$P(\text{survives}|\text{encounters k sites}) = (1-P_{ks})^{k} \qquad 4.33$$

To find the probability that the bomber survives the barrier SAM defense, condition on the number of sites it encounters

$$P_{BS} = \sum_{k} P(\text{survive}|\text{encounter k sites})P(\text{encounter k sites})$$

$$= \sum_{0}^{N_B}(1-P_{ks})^{k}\binom{N_B}{k}P^{k}(1-P)^{n-k}$$

$$= \sum_{0}^{N_B}\binom{N_B}{k}[P(1-P_{k})]^{k}(1-P)^{N-k}$$

$$= [P(1-P_{ks}) + (1-P)]^{N_B}$$

$$= [1 - PP_{ks}]^{N_B}$$

$$= [1 - (w/W)P_{k}]^{N_B} \qquad 4.34$$

as above.

Now let there be two forces of bombers, with B_h and B_ℓ denoting the numbers of bombers at high and low altitude respectively, approaching the barrier of SAMs. Assume that (a) within each force each bomber's probability of survival is the same, (b) each bomber's survival is independent of the survival of the other bombers, and (c) the forces are independent of each other. In this case, the distribution of the number of bombers that survive from each force is binomial.

$$P(\text{n survive from high force}|B_h \text{ in high force}) =$$

$$\binom{B_h}{n}P_{BSh}^{\,n}(1-P_{BSh})^{B_h-n} \qquad 4.35a$$

P(m survive from low force | B_ℓ is low force) =

$$\binom{B_\ell}{m} P_{BS\ell}^{m}(1-P_{BS\ell})^{B_\ell-m} \qquad 4.35b$$

where

$P_{BSh} - (1-(w_h/W)P_{ksh})^{N_B}$ and

$P_{BS\ell} = (1-(w_\ell/W)P_{ks\ell})^{N_B}$.

CHAPTER V

AREA DEFENSES

In this chapter, the model representation for the two forms of area defense will be developed. Specifically, these defenses are random area SAMs (RASAM) and fighter-interceptors (FI).

In each of the four parts of the model developed in the previous chapter, it was only necessary to know for each bomber whether it "succeeded" or "failed" that aspect of its mission. In the case of FAD, for example, the bomber either survived the entire engagement or was killed; if it was killed, it did not matter how long it had survived or how many intercepts it had survived before it was finally shot down.

The situation is different in the case of area defense. Now the bomber is at the stage where it will begin to release its weapons. Each bomber has been assumed to carry more than one weapon and the bomber's targets are located at different depths along the bomber's penetration route into the defended area. The bomber need not survive all the way through the defended area in order to release some or maybe all of his weapons. Rather, the number of weapons he releases, and hence the damage he extracts, will depend on how far into the defended area the bomber successfully penetrates.

For this reason, it is desired to find a probability of survival for a bomber that is a function of the bomber's depth of penetration. This penetration probability function will be denoted $P_p(x)$ and will incorporate the bomber's probabilities of successfully penetrating to at least a depth of x through both RASAM and FI defenses.

In this model, it is assumed that the RASAM and FI defenses operate independently of each other. This allows each defense to be modelled separately and the results combined to give the desired penetration probability function. The remainder of this chapter describes the modelling of the RASAM and FI engagements individually and finally derives the penetration probability function.

RASAM

The approach used to model RASAMs is very similar to that employed for the BSAMs. The major difference is that now the problem has depth and it is desired to find a probability of survival as a function of depth of penetration.

Consider the case of a single bomber flying a course due south down the corridor. As in the case of the BSAM problem, first look at the case where the entire corridor is defended by just one SAM site. The SAM site will be modelled using a probable lethal diameter (PLD) and a probability of kill, P_{ks}, within the PLD, consistent with the description in the BSAM section. The PLD and P_{ks} need not be the same for RASAMs as they were for BSAMs.

Three possible subcases of the scenario just described are illustrated in Figure 5.1, showing the defended territory, the bomber course and the SAM site. In Figure 5.1, the dashed lines represent a distance to either side of the bomber path equal to half the PLD of the SAM site.

The bomber's path will cross through the coverage of the SAM site only if the center of the SAM site is located between the two dashed lines in Figure 5.1. In other words, for a SAM site of coverage width w=PLD, to threaten the bomber, the site must be centered at a distance no greater than $\frac{1}{2}w$ from either side of the bomber path. Furthermore,

no sites south of the distance x which the bomber has penetrated to will have yet threatened the bomber.

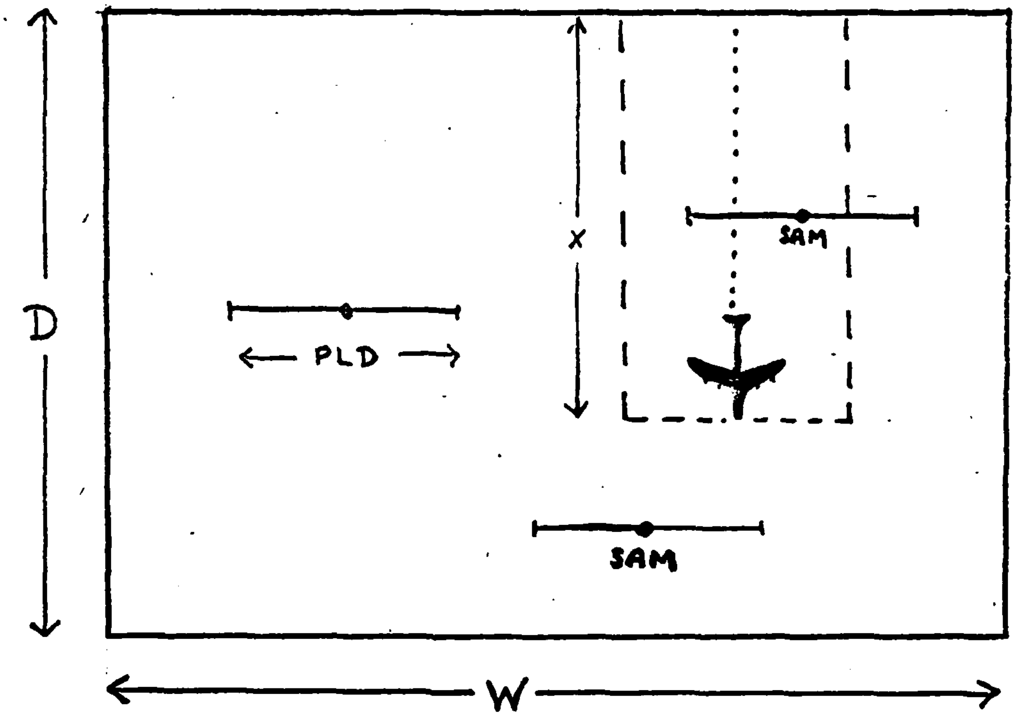

FIGURE 5.1

Bomber vs. random area SAMs.

Now consider the probability that the bomber crossed through the coverage of the SAM site while penetrating to a depth x into the defended territory. The bomber will have passed through the coverage of the SAM site if the center of the SAM site is located in the area between the two dashed lines. This area is A=wx. The total area in which the SAM site may be found is just the total defended area which is $A_T=W_D$. Since it is assumed that the SAM is located anywhere in A_T with uniform probability, the probability the SAM engages the bomber, P, is the ratio of the two areas or

$$P = \frac{wx}{WD} \qquad 5.1$$

If the bomber crosses through the coverage of the SAM site, its probability of being killed is P_{ks}. Therefore, the probability that the bomber survives to at least a depth x against one RASAM ($P_S'(x)$) is

$$P_x'(x) = 1-PP_{ks} = 1-P_{ks}\frac{wx}{WD} \qquad 5.2$$

Note that the probability that the bomber penetrates all the way through the defended area is

$$P_s'(D) = 1-P_{ks}\frac{w}{W} \qquad 5.3$$

which is the same result derived for the BSAM problem.

Now consider the case where there are N_R identical RASAMs defending the country. Assume (a) each of the SAMs is located independently of the others and (b) the engagements between the SAMs and bombers are independent. Therefore, if the bomber is to survive to a depth x, he must survive independent threats from all N_R SAMs. His probability of surviving to a depth x, against SAM defense is therefore

$$P_s(x) = \left(1-P_{ks}\frac{wx}{WD}\right)^{N_R} \qquad 5.4$$

Two implied assumptions should be noted about the above formulation of the RASAM problem. When deriving the probability that a bomber would cross through the coverage of a given SAM site, the area, wx, within which the SAM site had to be located in order to threaten the bomber was implicitly assumed to be entirely contained within the total defended area, WD. However, if the bomber's course is close to either border of the defended area, this may not be the case. Any bomber course less

than a distance $\frac{1}{2}w$ inside the boundary would place part of the area wx outside the defended territory. For such a bomber, the probability of crossing the coverage of a given SAM site would be slightly less than the probability given in equation 5.1. This is an improbable scenario; most targets will be more centrally located and the bomber's course will therefore not be that close to the boundary of the defended area.

Also, equation 5.1 implicitly assumes that a SAM site does not threaten a bomber until the bomber has reached a point even with the center of the SAM site. In principle, since SAM footprints are two-dimensional, it would be possible for the bomber to be killed by a SAM centered near the bomber's projected path and slightly south of the distance x, roughly between x and $x+\frac{1}{2}w$. To correct Equation 5.1 to account for this would require making further assumptions about SAM coverage. For values of x, W, and D much larger than w, the practical effect of such a correction would be negligible.

FIGHTER-INTERCEPTOR

At first glance, it might appear that the fighter-bomber engagement problem over land might be modelled similarly to the FAD problem. However, several things prevent such an approach. First, the model is concerned with finding a probability of survival as a function of the depth of penetration. There was no problem extending the BSAM model to include the depth dimension, thereby arriving at a model for RASAMs. However, unlike SAM sites, fighters are mobile and can cover much larger areas so the same method cannot be employed to extend the FAD model into an area FI defense model.

Another factor preventing any easy modification of the FAD approach is the problem of determining the number of intercepts that can be made

on each bomber. It is no longer reasonable to assume that each fighter will have only one chance to make an intercept. If the bombers are penetrating to depths measured in thousands of kilometers, the engagement will last long enough to allow fighters to make an intercept, return to the base, refuel, and then repeat the process, possibly several times.

For these reasons, an entirely different approach is used to model the bomber-FI engagement. The method is essentially that used in the SRI COPEM-1 model and is described below (Ref 5:27-33).

Before beginning the actual derivation, some variables must be defined.

$P_F'(t'|t_d)$ will denote the probability that a bomber that is detected at some time t_d after beginning penetration survives an additional time of at least t'.

$P_i(t'|t_d)$ will be defined as the probability that exactly i of the fighters that are assigned to the bomber detected at time, t_d, are able to intercept the bomber within a time t' after the time of detection t_d. This does not imply that all i fighters will successfully detect and convert on the bomber, only that the i fighters are close enough to the projected course of the bomber that they can be vectored into the vicinity of the bomber within time t'.

The probability that a fighter successfully detects, converts on, and kills the bomber it has been assigned to intercept will be denoted by P_{kf}.

$P_F''(t)$ will be probability that the bomber survives to a time t after initially entering the corridor and beginning penetration. (The

primes serve only to distinguish this function from other functions which will also be labelled P_F).

The probability distribution function (pdf) for the time after entering the corridor that the bomber is first detected by the radar net is $p_d(t_d)$.

The pdf for the time at which a bomber first enters the corridor will be given by $p_a(t_a)$.

Note the difference between t_d and t_a. The time of detection, t_d, is measured for each bomber from the time the bomber enters the corridor and does not specify a point in time of the engagement. The arrival time at the corridor entrance however is measured from some standard time 0 for all bombers. If the engagement is said to begin at time 0, when the first bomber begins penetration, then a subsequent bomber will be said to begin penetration at some time, t_a and be detected at some time t_d later, in other words at time t_a+t_d.

Consider first $P_s(t'|t_d)$, the probability that the bomber survives at least an additional time t' after being detected at t_d. This probability can be conditioned on the number of intercepts that occur within the time t' as follows:

$$P_F(t'|t_d) = \sum_{i=0}^{\infty} (1-P_{kf})^i P_i(t'|t_d) \qquad 5.5$$

Assume temporarily that the bomber begins penetrating at time t=0. $P_F''(t)$, the probability that a bomber survives to time t, can be found by conditioning on the time of detection, t_d.

$$P_F''(t) = \int_0^t P_s(t-t_d|t_d)p_d(t_d)dt_d + \int_t^{\infty} p_d(t_d)dt_d \qquad 5.6$$

The first integral expresses the probability that the bomber is detected at some time t_d less than t and then survives an additional time $t-t_d$. The second integral gives the probability that the bomber is not detected until some time greater than t, in which case it is assumed that he is guaranteed of surviving to time t. Equation 5.5 can be substituted into 5.6 to give

$$P_F''(t) = \int_0^t \sum_{i=0}^{\infty} (1-P_{kf})^i P_i(t-t_d|t_d)p_d(t_d)dt_d + 1 - \int_0^t p_d(t_d)dt_d$$

$$= \sum_{i=0}^{\infty} (1-P_{kf})^i \int_0^t P_i(t-t_d|t_d)p_d(td)dt_d + 1 - P_d(t) \qquad 5.7$$

where $P_d(t)$ is the cumulative distribution function (CDF) of $p_d(t_d)$ and the functions are all assumed to be well behaved so that the integration and summation can be exchanged.

Equation 5.7 gives a bomber's probability of survival to at least a time t. The quantity that is of interest to the model is a function $P_F'(x)$ which gives the probability of survival for the bomber as a function of penetration depth. This can be found by assuming that the bombers fly at a constant velocity v. In a time t then, the bomber will travel a distance x=vt. Put differently, in order to survive to a distance x, a bomber must survive a time t=x/v. Thus,

$$P_F'(x) = P_F''(x/v) = \sum_{i=0}^{\infty} (1-P_{kf})^i \int_0^{x/v} P_i(\frac{x}{v} - t_d|t_d)p_d(t_d)dt_d$$

$$+ [1-P_d(\frac{x}{v})] \qquad 5.8$$

The last thing to account for is the possibility that the bomber enters the corridor at some time greater than 0. It is assumed that

the probability of detection depends only on the depth of penetration and not on the time of arrival at the corridor entrance. The function $P_i(t'|t)$, however, may in general depend on the absolute time at which detection occurs. For example, $P_i(t'|t)$, the probability that i intercepts could take place within time t' after detection of the bomber at time t, may depend on either the number of fighters or the number of bombers in the air at time t. These numbers would both vary with absolute time, not with the time of detection t_d of a bomber. Thus,

$$P_i(t'|t) = P_i(t'|t_a+t_d)$$

instead of simply

$$P_i(t'|t) = P_i(t'|t_d)$$

By making this one change in Equation 5.8, one can find a function $P_F(x|t_a)$ which is the probability a bomber survives to penetrate at least a distance x given that he begins penetration at time t_a;

$$P_F(x|t_a) = \sum_{i=o}^{\infty} (1-P_{kf})^i \int_o^{x/v} P_i(\tfrac{x}{v} - t_d|t_a+t_d)p_d(t_d)dt_d + [1 - P_d(\tfrac{x}{v})] \qquad 5.9$$

Typically, one is not concerned with the order in which bombers penetrate. The unconditional average penetration probability, $P_F(x)$, is

$$P_F(x) = \int_o^T P_F'(x|t_a)p_a(t_a)dt_a$$

$$= \int_o^T \{ \sum_{i=o}^{\infty} (1-P_{kf})^i \int_o^{x/v} P_i(\tfrac{x}{v} - t_d|t_a+t_d)p_d(t_d)dt_a + [1-P_d(\tfrac{x}{v})]\}p_a(t_a)dt_a$$

$$= \sum_{i=o}^{\infty} (1-P_{kf})^i \int_o^T \int_o^{x/v} P_i(\frac{x}{v} - t_d | t_a + t_d) p_d(t_d) p_a(t_a) dt_a dt_a$$

$$+ [1-P_d(\frac{x}{v})] \qquad 5.10$$

Once again, all the functions have been assumed to be well behaved so that the integral and the summation can be exchanged.

This is the general form of the penetration probability function derived by de Sobrino, Monahan, Weinstein, and Yu. (Ref 5:27-30). The next step is to identify choices for the functions $P_i(t'|t)$, $p_d(t_d)$ and $p_a(t_a)$.

Consider first the function $P_i(t'|t)$ which is the probability density function for the number of possible intercepts that may be made on a bomber within a time t' after detection at time t. One way of looking at the problem is to consider a counting function N(t') which counts the number of events, in this case intercepts, that occur in the time interval [0,t'].

Since it has been assumed that bombers are detected by a ground based radar net and not by chance encounters with airborne interceptors, it is reasonable to assume that

$$N(0) = 0 \qquad 5.11$$

In other words no intercepts will have been made at the time of detection.

In addition, assume that events cannot occur simultaneously; some time, no matter how small, must elapse between intercepts. Mathematically,

$$\text{For any } t \geq 0, \quad \lim_{h \to o} \frac{P[N(t+h)-N(t) \geq 2]}{P[N(t+h)-N(t)=1]} = 0 \qquad 5.12$$

Consider now a third assumption:

$$\text{For any } t>0, \quad 0<P[N(t)>0]<1 \qquad 5.13$$

This assumption implies that in any time interval, regardless how small, there is a finite probability that at least one intercept could occur but there is no certainty that an event will occur.

Detection of a bomber does not guarantee that there will be an unassigned fighter available to intercept the bomber. There is a chance, in other words, that even though the bomber is detected, there will be no intercepts made on it. The upper inequality in Equation 5.13 therefore seems reasonable.

The lower inequality poses more of a problem. Physical limitations of the command and control system will cause some time lag, maybe quite small, between the actual detection of the bomber and assignment of an interceptor to it. Even if instantaneous communications were possible, some time would still have to elapse to allow the fighter to reach its interception point with the bomber. Therefore, it seems likely that $P[N(t)>0]=0$ for some small values of $t, 0<t<\varepsilon$, contrary to the assumption in Equation 5.13. Nevertheless, allowing extremely fast communications may not be too unreasonable and one can imagine a case where an interceptor would be stationed quite close to the place where the bomber was detected. The ε, which would describe the absolute minimum time that would elapse before the first intercept was possible, might then be quite small. Assuming that $\varepsilon=0$ would give the defense only a

very small advantage and should not significantly affect the results of the model.

Equations 5.11-13 are three of the four assumptions that define a Poisson process. The fourth assumption states that the process has independent increments. Mathematically, this says that for all choices of t_i such that $0<t_1<t_2 \ldots <t_n$, the n random variables $[N(t_1)-N(t_0)]$, $[N(t_2)-N(t_1)]$, ..., $[N(t_n)-N(t_{n-1})]$ are independently distributed.

In this model the above assumption would imply that the distribution of the number of intercepts that could occur in (t_1, t_2) would be independent of the number that had occurred in $(0,t_1)$. This assumption is clearly not very sound for small numbers of interceptors. Imagine the case where there is only one fighter. If it made an intercept in the interval $(0,t_1)$ then it might be impossible for it to make another intercept in (t_1,t_2) because it had to return to base. This would violate the assumption. However, as the number of interceptors increases, this becomes less of a problem. Thus, for reasonably large numbers of interceptors the assumption is probably acceptable. If the number of interceptors is not too large, which might be the case if the bomber strike were preceded by a nuclear missile strike, then this assumption may be very offense conservative.

If all four of these assumptions are accepted, keeping in mind their limitations, then one can claim that the intercept probability, $P_i(t'|t)$, may be approximated with a Poisson distribution:

$$P_i(t'|t) = (\lambda_t t')^i e^{-\lambda_t t'}/i! \qquad i = 0, 1, 2, \ldots \qquad 5.14$$

This is the function used by deSobrino, Monahan, Weinstein and Yu in the COPEM-1 model (Ref 5:30-33). The paramter λ_t is called the

intercept intensity function and is subscripted to indicated that it may depend on time. For a bomber detected at time t, λ_t is the average number of intercepts possible against the bomber per unit time. In a time t' after detection of the bomber, the expected number of possible intercepts is λ_t.

Substituting Equation 5.14 into Equation 5.10, one has

$$P_f(x) = \sum_{i=o}^{\infty} (1-P_{kf})^i \int_o^T \int_o^{x/v} \frac{\lambda_{(t_a+t_d)}(\frac{x}{v} - t_d)^i e^{-\lambda_{(t_a+t_d)}(\frac{x}{v}-t_d)}}{i!}$$

$$p_d(t_d)p_a(t_a)dt_a dt_a + [1-P_d(\tfrac{x}{v})] \qquad 5.15$$

By re-exchanging the summation with both integrals and using the identity

$$\sum_{i=o}^{\infty} \frac{x^i}{i!} = e^x \qquad 5.16$$

Equation 5.15 can be reduced to

$$P_F(x) = \int_0^T \int_0^{x/v} \exp[-P_{kf}\lambda_{(t_a+t_d)}(\tfrac{x}{v} - t_d)p_d(t_d)p_a(t_a)dt_d dt_a$$

$$+ [1-P_d(\tfrac{x}{v})] \qquad 5.17$$

Leaving aside the pdfs of t_a and t_d temporarily, Equation 5.17 still has the problem of a choice for the function λ_t. This function will in general depend on such factors as the number and location of airborne, unassigned interceptors, interceptor characteristics such as velocity, and the number of penetrators in the defended area. At any time, the function will also depend on the events that have occurred

earlier in the engagement. de Sobrino et.al. have developed an iterative routine that can be used to approximate this function. The concept is based on properties of the Poisson distribution. For a Poisson distribution with parameter λ, the waiting time to the first event is distributed exponentially with the same parameter λ. The expected value of the exponential deistribution is $1/\lambda$. Therefore, if an estimate can be found for the expected waiting time to the first intercept of a bomber detected at time t, the reciprocal of this estimate will in turn be an estimate of the parameter λ (Ref 5:33).

A description of the calculation to find the expected waiting time to the first intercept can be found in references 4 and 8. This calculation can be performed for several values of t and then appropriate interpolation procedures can be employed to give an estimate of λ_t for other times. This estimate can then be used to numerically integrate Equation 5.17 (assuming suitable functions or estimates of $p_a(t_a)$ and $p_d(t_d)$ have been specified).

This approach probably gives the best, in the sense of most intuitively appealing, results. However, if just two additional assumptions are made, a vast simplification results on Equation 5.17.

First consider what happens if the parameter λ is assumed to be independent of time. In this case, one has

$$P_F(x) = \int_0^T \int_0^{x/v} \exp[-P_{kf}\lambda(\frac{x}{v} - (t_d)] p_d(t_d) p_a(t_a) dt_d dt_a$$

$$+ [1-P_d(\frac{x}{v})] \qquad 5.18$$

The integrand of Equation 5.18 no longer contains any terms which depend on t_a except the pdf of t_a. Since the integral of a pdf over its full range is 1, one has

$$P_F(x) = \int_0^{x/v} \exp[-P_{kf}\lambda(\frac{x}{v} - t_d)]p_d(t_d)dt_a+[1-P_d(\frac{x}{v})] \qquad 5.19$$

With λ assumed to be a (still unspecified) constant, the only function of time remaining in the integral is the pdf of t_d, the distribution of the time of detection of the bomber. Consider this function now.

It has already been assumed that the bombers' probability of detection is a function only of its altitude and depth of penetration. Temporarily assume that all the bombers are flying at the same constant altitude. Now one is left with a function depending only on the depth of penetration which, since the bombers are also assumed to be flying at a constant velocity, is equivalent to being a function of penetration time.

Assume that the defensive ground based detection system uniformly covers the defended area. If this is the case, then it is reasonable to believe that an undetected bomber's chance of being detected while flying a given distance, or equivalently a given time, would be the same for all distances of the same length no matter where they were along the bomber's path. For example, consider a bomber that has penetrated undetected to a depth of 1,000 kilometers. His probability of escaping detection for the next 100 kilometers will be the same as his original probability of penetrating the first 100 kilometers undetected.

Mathematically, using time instead of distance, the above can be expressed as

$$P[(t_d>t_1+t_2)|t_d>t_1] = P(t_d>t_2) \qquad 5.20$$

where t_d is the time of detection after penetration has begun.

Equation 5.20 is a property of the exponential distribution. If one is willing to assume that the distribution of the time to detection of a bomber is characterized by the above assumptions, then,

$$p_d(t_d) = \alpha e^{-\alpha t_d} \quad \alpha>0,\ t_d\geq 0 \qquad 5.21$$

The parameter α is a measure of the capability of the radar net to detect the bomber. Specifically, $1/\alpha$ is the expected time to detection of the bomber. An estimate of the expected time to first detection should consider the intensity of the defensive radar coverage and the effectiveness of bomber tactics such as low level penetration.

Substituting Equation 5.22 into Equation 5.18, one has

$$P_F(x) = \int_0^{x/v} \exp[-P_{df}\lambda(\frac{x}{v} - t_d)]\alpha e^{-\alpha t_d}dt_d+[1- \int_0^{x/v}\alpha e^{-\alpha t_d}dt_d] \qquad 5.22$$

where the CDF of t_d has been expressed as the integral of the pdf in the last term.

The above function can now be integrated directly to give, after combining terms,

$$P_F(x) = (\frac{P_{kf}\lambda}{P_{kf}\lambda-\alpha})e^{-\alpha\frac{x}{v}} - (\frac{\alpha}{P_{kf}\lambda-\alpha})e^{-P_{kf}\lambda\frac{x}{v}} \qquad 5.23$$

This function has the major advantage of simplicity. No numerical integrations are required. A bomber's probability of penetrating to at least a distance x inside the defended area is now expressed as a simple function of its velocity and three other parameters. Because

of this simplicity, this is the form of the penetration probability against FI defenses that is used in this model.

As an input, the paramter P_{kf}, the probability that a fighter kills the bomber it is assigned to, can be estimated using the characteristics of the aircraft and models of the engagement such as COLLIDE (Ref 1:B-5). Reasonable choices for the parameters α and λ, however, present greater difficulties. One approach to the problem of estimating λ has already been referenced. It would seem reasonable that a similar method could be developed to estimate α. Explicit techniques for estimating these parameters is a suggested future research topic. For the purposes of this thesis, both α and λ, as well as P_{kf}, will be taken to be externally provided to the model.

PENETRATION PROBABILITY FUNCTION

The previous two sections have derived functions, $P_S(x)$ and $P_F(x)$, giving a bomber's probability of surviving to at least a depth x against either SAM defenses or FI defenses individually. The only thing that remains to do is to combine these results into a single penetration probability function $P_p(x)$.

In order for a bomber to sucessfully penetrate to a depth x, he must survive the threats from both forms of defense. It has been assumed that the two defenses effect the bomber's survival independently. Thus, his probability of surviving both defenses is just the product of his probability of surviving each defense individually. Therefore, one has

$$P_p(x) = P_S(x)P_F(x) \qquad . \qquad 5.24$$

Substituting from Equations 5.4 and 5.23 for the functions $P_s(x)$ and $P_f(x)$, one gets

$$P_p(x) = [1-P_{ks}\frac{wx}{WD}]^{N_R}[(\frac{P_{kf}\lambda}{P_{kf}\lambda-\alpha})e^{-\alpha\frac{x}{v}} - (\frac{\alpha}{P_{kf}\lambda-\alpha})e^{-P_{kf}\lambda\frac{x}{v}}] \quad . \qquad 5.25$$

This is the final form of the penetration probability function.

Up to this point, no distinction has been made in the derivation between bombers flying at high and low altitudes. The reasoning will be identical in both cases and the functional form of the penetration probability function will thus be the same for both. Various performance parameters (P_{ks}, P_{kf}, w, v, α and possibly λ) may have different values when the bombers penetrate at high altitudes than when bombers penetrate at low altitudes. Therefore, the model will use two different probability penetration functions corresponding to the two sets of values for the parameters.

CHAPTER VI

MISSION COMPLETION AND CRUISE MISSILES

In this chapter, the model descriptions for the last two stages of the bomber mission are developed. Specifically, these stages are weapons delivered, including the terminal SAM defenses, and recovery.

Also in this chapter the model representations for cruise missiles (CM) are described. Three different cases will be covered corresponding to CMs of three different ranges.

TERMINAL SAMs AND WEAPONS DELIVERED

The next problem to be addressed is the modelling of terminal SAM engagements and the number of weapons delivered.

Consider first the case of one bomber and its load of N weapons. It has already been assumed that the weapons are homogeneous and that only one weapon will be expended on each target. The bomber's targets are assumed to have been preselected as part of his mission plan and are located at various distances along his course into the defended area. The targets are all assumed to be uniformly defended by some number, possibly zero, of terminal SAMs.

Label the points at which the bomber's targets are located x_1, x_2, ..., x_n, where $o < x_1 < x_2 < \ldots.. < x_n < D$. The distances are measured from where the bomber began penetration and D is the total depth of the defended area.

Let n be the number of weapons released. The probability that at least one weapon is released may be written $P(n \geq 1)$. For one weapon to be released, the bomber must have penetrated to at least the depth of his first target x_1. This probability that the bomber survives as a

function of the depth of penetration has been derived in Chapter V and is given by the penetration probability function $P_S(x_1)$.

Besides reaching the target through the area defenses, it is assumed that the bomber must survive the terminal SAM defenses before he can release a weapon at that target. Denote this probability of survival as P_T. Since the targets are assumed to be uniformly defended P_T will be a constant in the model for bombers flying at a given altitude. P_T need not be the same, however, for bombers, at different altitudes. Note also that P_T does not have to be the same as the bomber's chance of surviving a SAM attack in general. Several different assumptions can be made about terminal SAM defenses and the offensive tactics used against them. As just one example, consider targets defended by fixed SAM sites. The bombers may attack the SAM sites with SRAMs before moving in closer to release a weapon at the target itself. In this scenario, P_T would be the same as the probability that the bomber successfully destroys the SAM site or fails to destroy it but survives the SAM attack. In the case of undefended targets P_T is set to 1.

From the above, the probability that at least one weapon is released is

$$P(n \geq 1) = P_S(x_1)P_T \qquad . \qquad 6.1$$

The probability that at least two weapons are released can be found in the same way. To have released two weapons, the bomber must penetrate to at least a depth x_2 and must have survived two terminal SAM engagements, one at each target. Hence,

$$P(n \geq 2) = P_s(x_2)P_T^2 \qquad 6.2$$

In general, one finds

$$P(n \geq i) = P_s(x_i)p_T^i \quad i = 0, N; \quad x_o \equiv o \qquad 6.3$$

Now consider the probability that exactly one weapon is released. Since the number of weapons released is a discrete distribution one has

$$P(n = 1) = P(n \geq 1) - P(n > 1)$$

$$= P(n \geq 1) - P(n \geq 2) \qquad 6.4$$

Therefore the probability that exactly one weapon is released can be expressed

$$P(n = 1) = P_s(x_1)P_T - P_s(x_2)P_T^2 \qquad 6.5$$

and in general for exactly i weapons, one has

$$P(n = i) = P_s(x_i)P_T^i - P_s(x_{i+1})P_T^{i+1} \qquad i = 0, N-1 \qquad 6.6a$$

$$P(n = N) = P_s(x_N)P_T^N \qquad 6.6b$$

Since the bomber only carries N weapons the probability of releasing more than N is zero or, to put it differently, the probability of releasing exactly N is the same as the probability of releasing at least N. Thus the difference between equations 6.6a and 6.6b.

Equations 6.6 give the distribution for the number of weapons released by each bomber. For a given number of bombers that survived the BSAM defense to begin penetration, it would in principle be

possible to find a distribution for the total number of weapons released by all (say B) bombers. This would be given by

$$P(n_t=k) = \sum_{i_1=o}^{N} \sum_{i_2=o}^{N} \sum_{i_{B-1}=0}^{N} P\_(n_1=i_1)P(n_2=i_2)\cdots\cdot P(n_{B-1}-i_{B-1})$$

$$P(n_N=k-i_1-i_2 \cdots\cdot - i_{B-1}) \qquad 6.7$$

Where n_t is the total number of weapons releases and n_i is the number of weapons released from the i^{th} bomber and $P(n_t < 0) = P(n_t > N) = 0$. For large numbers of surviving bombers, this would be a very unweildy sum. An approximate alternative method is outlined below.

Consider a single arbitrary weapon out of the N weapons on a particular penetrating bomber. Since this weapon is identical to all the others on the bomber, it will not be assigned to a particular target, rather it will have equal probability of being released over any of the targets. Another way of looking at that is that as the bomber reaches any given target, that weapon is as likely to be released as any other weapon.

If the bomber survives to release a weapon over exactly one target, the weapon will have a chance $P_1 = 1/N$ of being the one that is released. Similarly, if the bomber survives to attack exactly two targets, the weapon will have a probability $P_2 = 2/N$ of being one of the two weapons released and in general the weapon's probability of being released if the bomber succeeds in attacking i targets is $P_i = i/N$.

By conditioning on the number of weapons released by the bomber, the probability that the given weapon is released, denoted P_r, is

$$P_r = \sum_i P(\text{release} \mid i \text{ weapons released}) P(i \text{ weapons released})$$

$$= \sum_{i=o}^{N} (i/N) P(n=i)$$

$$= (1/N) \sum_{i=o}^{N} i \, P(n=i)$$

$$= (1/N) E(n) \qquad 6.8$$

where E(n) is the expected number of weapons released by the bomber.

The number of bombers that survive the BSAM defense is known (B_p) and hence the total number of weapons that may be released is known (NB_p) (since the bombers are assumed to have equal weapon loads). Assume that each bomber's set of targets is located at the same distances x_i along its penetration path. Then the probability of being released over a target as given by equation 6.8 is the same for all weapons. If it is further assumed that each weapon's release event is independent of what has happened to the other weapons (discussed later), then the distribution of the total number of weapons released is binomial:

$$P_r(k|B_p) = \binom{BN_p}{K} P_r^k (1-P_r)^{NB_p-k} \qquad o \leq k \leq B_p N \qquad 6.9$$

where B is the number of penetrating bombers that have survived the BSAMs each carrying N weapons.

The distribution in Equation 6.9 just gives a distribution for the number of weapons released. The distribution of the number of targets actually destroyed will not be the same unless each weapon that is released has a probability 1 of hitting and destroying its target. It is possible for the weapon to malfunction, or, if the target is

extremely hard, the weapon may not impact close enough to the target to destroy it. If q is the probability that a weapon that is released will destroy its target, then the probability that a weapon is released and destroys its target, P_d, is

$$P_d = P_r q \qquad 6.10$$

and the distribution of the number of targets destroyed is given by

$$P_d(k' b_p) = \binom{NB_p}{k} P_d^k (1 - P_d)^{NB_p - k} \qquad 6.11$$

Two of the assumptions that have been made require further discussion. The assumption that each weapon's probability of being released is independent of what has happened t all the other weapons is not strictly accurate. If it is known that a specific bomber has already released half its weapons, then one would expect to find an increased probabiltiy of being released for any weapon still aboard. However, it may be reasonable to assume that the probability of being released for any weapon on one bomber would be independent of the probability of being released for any weaon on any other bomber. In the case where each bomber carries only one weapon, all the weapons will be independent and the binomial distribution given above will accurately describe the total number of weapons released. When there is more than one weapon on each bomber, then for reasonably large numbers of bombers most of the weapons will still be independent of each other and the binomial distribution given above should be a good approximation to the total number of weapons released.

The other assumption that needs clarification is the assumption that each bomber's set of targets is located at the same depth x_i

AD-A115 567 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 19/5
AN ANALYTIC MODEL OF THE STRATEGIC BOMBER PENETRATION MISSION W--ETC(U)
DEC 81 G P CLEMENS
UNCLASSIFIED AFIT/GOR/MA/81D-3 NL

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along the bomber's penetration path. Strictly interpreted, this assumption implies that all the targets in the defended area are located on N distinct lines of latitude and that there are the same number of targets on each line. In reality, of course, the targets will be distributed over all depths of penetration. The problem is to choose values of x_i so that the assumed target distribution is a reasonably close model of the real one. A method to do this is outlined below.

One possible target distribution is shown in Figure 6.1. Targets are denoted by T's. The targets are not distributed uniformly; more targets are shown in the middle parts of the defended area than in the northern or southern extremes. This seems reasonable but is not a necessary assumption; any distribution would work.

The target map in Figure 6.1 shows forty-eight targets. Assume those targets are to be attacked by a force of eight bombers each carrying six weapons. The map can be partitioned into horizontal bands each containing eight targets. This is done in Figure 6.2. Now each bomber can be assumed to release one of its weapons over one target in each of the six bands. The only thing that needs to be done is to choose a value of x_i for each band that will approximate the depth of all eight targets within the band. One choice would be to assume that all targets within a band are on the midline of the band. A more offense conservative choice is to assume that all targets in a band are on the southern boundary of the band.

An alternative method is available if it is assumed that the analyst knows the distribution of targets as a function of penetration depth. Consider a pdf that gives a random target's probability density

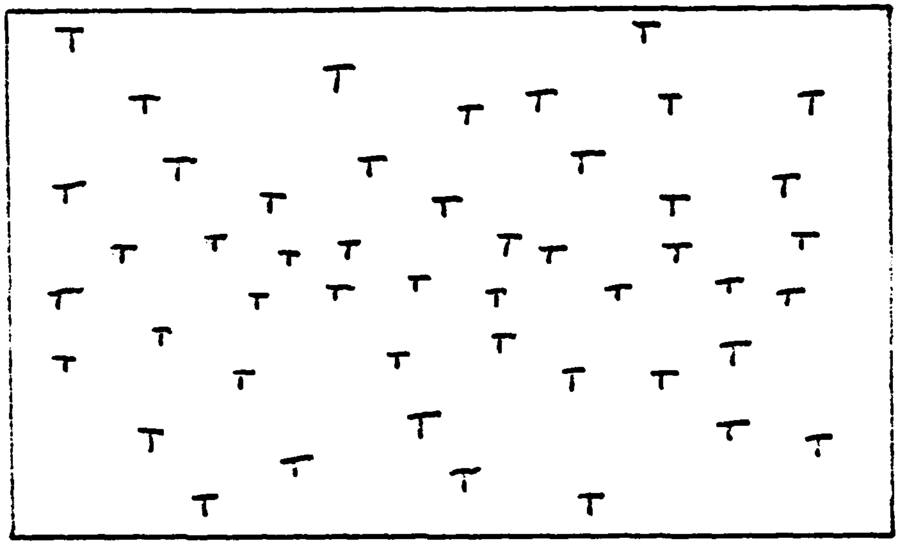

FIGURE 6.1

Distribution of targets within defended area.

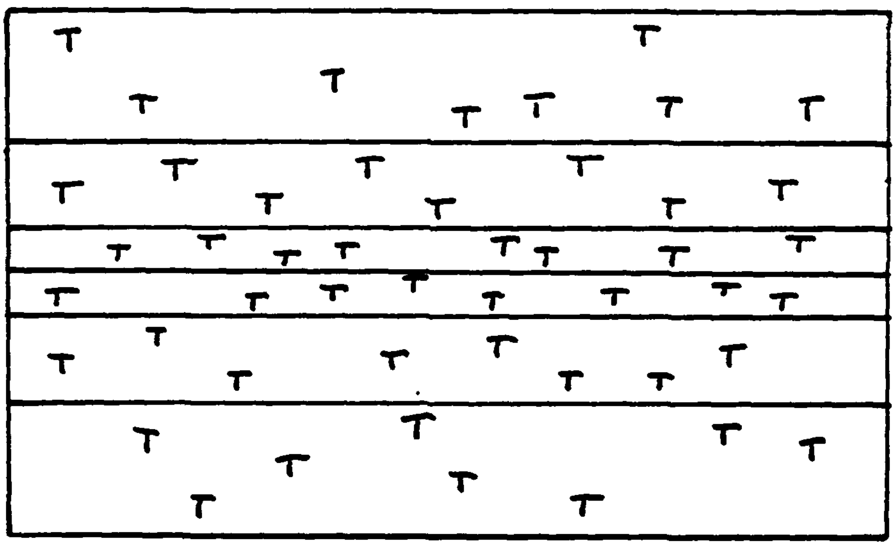

FIGURE 6.2

Target set partitioned into six bands.

of being located at a depth x. This probability might follow a normal, uniform, or any other distribution one wished to assume. A possible pdf is given in Figure 6.3.

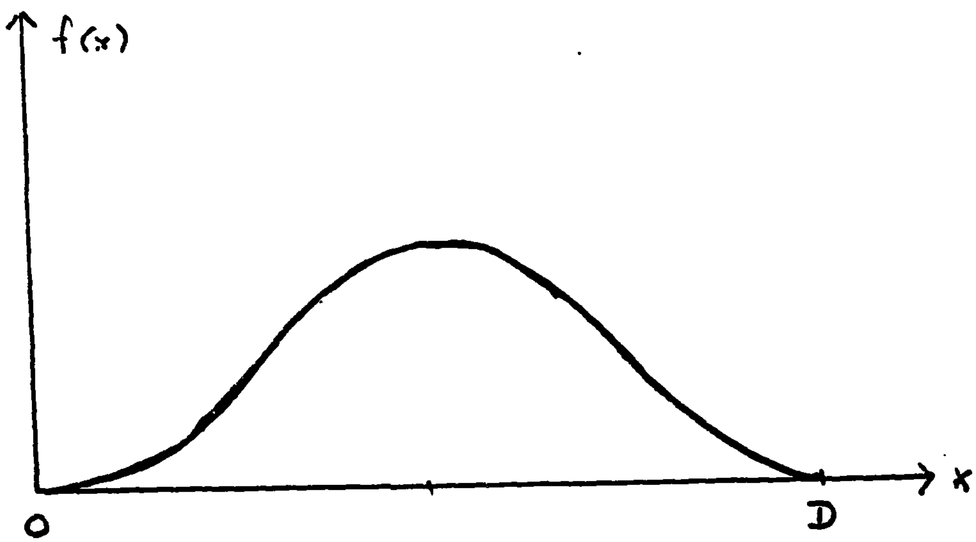

FIGURE 6.3

Probability distribution of targets as a function of depth.

This pdf can be partitioned into six intervals of equal area, corresponding to bands with equal probabilities, 1/6, that the target would be within each band. If the intervals are (0,a), (a,b), ···, (e,D), then the values of a, ..., e will be those values such that

$$\int_0^a p(x)dx = \int_a^b p(x)dx = \ldots = \int_e^D p(x)dx \qquad 6.12$$

The resulting partition is sketched in Figure 6.4. Since each of the intervals defines a band within which a target has equal

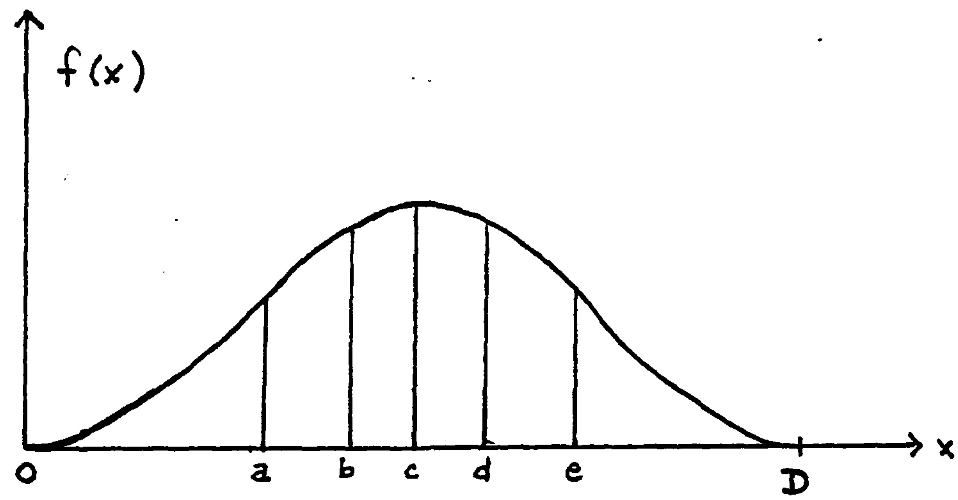

FIGURE 6.4

Target pdf partitioned into eight equal parts.

probability of being located, then for large numbers of targets that follow the assumed distribution, one could expect to find equal numbers of targets within each band. As before, it still remains to choose a value of x_i for each band. Various procedures exist that could be used to generate values that are mathematically best under various criteria; these will not be discussed here. For many applications, either of the simple choices mentioned previously would be adequate.

In this computer model, the x_i are assumed to be input, the model calling for as many x_i as there are weapons on each bomber.

It was pointed out earlier in this section that P_T need not be the same for bombers flying at high and low altitudes. The same applies to $P_p(x)$. Therefore, there will be two distributions for the number of weapons that destroy targets, corresponding to each of the two groups of bombers. These distributions can be combined to give the distribution for the total number of weapons delivered (see Chapter VII).

RECOVERY

The only aspect of the recovery problem to be addressed by this model is the bomber's problem of leaving the defended area after releasing all its weapons.

In this model, it is being assumed that after attacking its last assigned target, a bomber will continue to penetrate the rest of the way through the corridor and exit at the bottom or southern end. Using the penetration probability function $P_p(x)$, the bomber's chance of surviving the area defenses to reach the end of the defended area is P_s=P_s(D) where D is the total depth of the defended area.

It is assumed that if the bomber has reached the end of the defended area, he has already attacked all his assigned targets and survived the terminal defenses of each. The probability that a bomber that has survived the BSAM defense to begin penetration will survive to exit the defended area, P_e, is, therefore,

$$P_e = P_p(D)P_T^N \qquad 6.13$$

where N is the number of targets he attacked during penetration.

Assume that each bomber's recovery event is independent of every other bomber's recovery event. Then the distribution of the number of bombers that exit the defended area for a given number B_p of bombers that survive the BSAM defense is binomial.

$$P_R(k|B_p) = \binom{B_p}{k} P_e^k (1 - P_e)^{B_p - k} \qquad 6.14$$

Since both $P_p(x)$ and P_T may have different values for bombers flying at high and low altitudes, there will be two recovery distributions, one for each force of bombers.

CRUISE MISSILES

This section will develop the representation for cruise missiles (CM) under each of the three cases discussed in Chapter III. Specifically, these cases are short range cruise missiles (SRCM) that are released at various points during penetration after the bombers have passed through the BSAM band, intermediate range cruise missiles (IRCM) which are released after the FAD zone but before the bombers encounter the BSAM defense, and long range cruise missiles (LRCM) which can be released before the bombers enter the FAD zone.

Before discussing each of the three cases, some general assumptions will be made about the effect CMs would have on a bomber's penetration probabilities that have been discussed in the last three chapters.

The first assumption to be made is that introduction of CMs will not effect a bomber's probability of penetrating through BSAM or RASAM defenses. This assumption is reasonable in cases where there are insufficient penetrators to saturate the SAM defenses. As the number of penetrators becomes larger, it would be possible to degrade the

defenses simply by exhausting the supply of missiles at the SAM sites. For large total numbers of penetrators, an individual penetrator's chance of surviving the SAM defenses would tend to increase and the assumption would become quite offensive conservative.

The next assumptions deal with the effect of CMs on a bomber's probability of surviving the area fighter-interceptor defenses. Recall that this probability depends on a parameter λ which gives the average number of intercepts possible on a penetrator per unit time. It has been assumed in this model that λ is a constant. It is now further assumed that λ remains a constant when CMs are added to the problem and that λ is the same for both bombers and cruise missiles. This does not imply that λ must be the same constant for a model with CMs added as it was for a model without CMs, only that it is constant within each model and that it is the same for both CMs and bombers.

The last area in which CMs could effect a bomber's probability of survival is the FAD. If LRCMs have been released and are entering the FAD zone with the bombers, it is assumed that the defense will try to attack them as well as the bombers. Since there are a fixed number of defensive interceptors, each capable of attacking only one penetrator, an increase in the number of penetrators will decrease the number of interceptors assigned to each penetrator. This will increase each individual penetrator's probability of surviving the FAD. This is accounted for in this model.

Short Range Cruise Missiles. Consider a cruise missile of range R destined for a target at some distance x+R inside the defended area.

bomber will release the CM at its maximum range, the CM will be released when the bomber has penetrated to a dpeth x inside the defended area. In order for the CM to be released, the bomber carrying it must survive to at least the depth x. The probability of this is given by the penetration probability function derived in the last chapter.

$$P_{pb}(x) = [1-P_{ksb}\frac{w_b x}{WD}]^{N_R}[(\frac{P_{kfb}\lambda}{P_{kfb}\lambda-\alpha_b})e^{-\alpha_b x/v_b}-(\frac{\alpha_b}{P_{kfb}\lambda-\alpha_b})$$

$$e^{-P_{dfb}\lambda x/v_b}] \qquad 6.15$$

Recall that the term inside the second set of square brackets in Equation 6.15 is $P_F(x)$, the probability that a bomber survived to at least a depth x through FI defenses. The last step of the derivation of $P_F(x)$ in Chapter V was the integration of Equation 5.23.

$$P_F(x) = \int_0^{x/v_b} e^{-P_{kfb}\lambda(\frac{x}{v_b}-t_d)}\alpha_b e^{-\alpha_o t_d}dt_d+[1-\int_0^{x/v_b} x_b e^{-\alpha_b t_d}dt_d]$$

$$= (\frac{\alpha_b}{P_{kfb}\lambda-\alpha_b})(e^{-\alpha_b x/v_b}-e^{-P_{kfb}\lambda x/v_b})+e^{-\alpha_b x/v_b} \qquad 6.16$$

If the terms in equation 6.16 are combined, one will find that the expression is equivalent to the final form of $P_F(x)$ given in equation 5.24 and used in equation 6.15. However, in the expression for $P_F(x)$ in equation 6.15, each term has an interpretation in terms of the model. The first set of square brackets in equation 6.15 gives the probability that the bomber is detected by the defense but still survives to at least a depth x. The term in the second set of square backets is the probability that the bomber has reached a depth x

undetected. (See equation 5.6 and the accompanying text). These interpretations will be useful in the development of the SRCM model and for this reason equation 6.14 is rewritten

$$P_{Pb}(x) = [1-P_{ksb}\frac{w_b x}{WD}]^{N_R}[(\frac{\alpha_b}{P_{kfb}\lambda-\alpha_b})(e^{-\alpha_b x/v_b}-e^{-P_{kfb}\lambda x/v_b})]$$

$$+ [1-P_{ksb}\frac{w_b x}{WD}]^{N_R} e^{-\alpha_b x/v_b} \qquad 6.17$$

where equation 6.15 and the distributive property have been used.

Return now to the cruise missile. If it is successfully released at a depth x inside the defended area, it must then penetrate a further distance R on its own. Since it is assumed that the CM is threatened by the same defenses that threaten the bombers, it is reasonable to use the same general analysis that was developed in Chapter V for bomber penetration through area defenses. The only modifications will be to account for the fact that at the time of its release, the CM is already some distance into the defended area.

As has already been suggested by equation 6.16, there are two possible situations at the time the bomber releases the CM. First, the bomber may have penetrated to the release point undetected. The probability of this event, call it $P_x(u)$ has already been shown to be the second term on the RHS of Equation 6.17.

$$P_x(u) = [1-P_{ksb}\frac{w_b x}{WD}]^{N_R} e^{-\alpha_b x/v_b} \qquad 6.18$$

It seems reasonable to believe that if the bomber has not been detected at the time of release of the CM, then the CM itself will begin its flight undetected. In this case, the analysis that the CM

will penetrate an additional depth R on its own will parallel exactly the analysis given in Chapter V for bombers.

The analysis in Chapter V of bomber penetration through RASAMs gave a probability of bomber survival $P_s(x)$ that depended on the depth of penetration x into the defended area. The analysis assumed that x was measured from the northern boundary of the defended area since there was no way a bomber could begin penetration at any point south of the boundary. However, a cruise missile, because it is being released from a bomber which has already penetrated to a depth x, begins its own flight at a distance x inside the boundary. Nonetheless, using the same assumptions and analysis, it can be shown that the CMs probability of penetrating a distance R from its release point x is a function only of the distance R, not x. This is because the analysis used a ratio of two areas to find the probability of encountering a particular SAM site. The area in the denominator was just the area of the entire defended area. The area in the numerator depended on the length of the penetration path and the PLD of a SAM site; this would be the same for a path of given length no matter where the path began. See Figure 6.5.

The probability that a cruise missile penetrates a distance R through RASAM defenses is therefore

$$P_{sc}(R) = [1-P_{ksc}\frac{w_c R}{WD}]^{N_R} \qquad 6.19$$

Now consider the FI defenses. Since the parameter λ is assumed to be a constant, the number of intercepts that are possible against the cruise missile depends only on the amount of time from first detection of the CM until it reaches its target. The time until

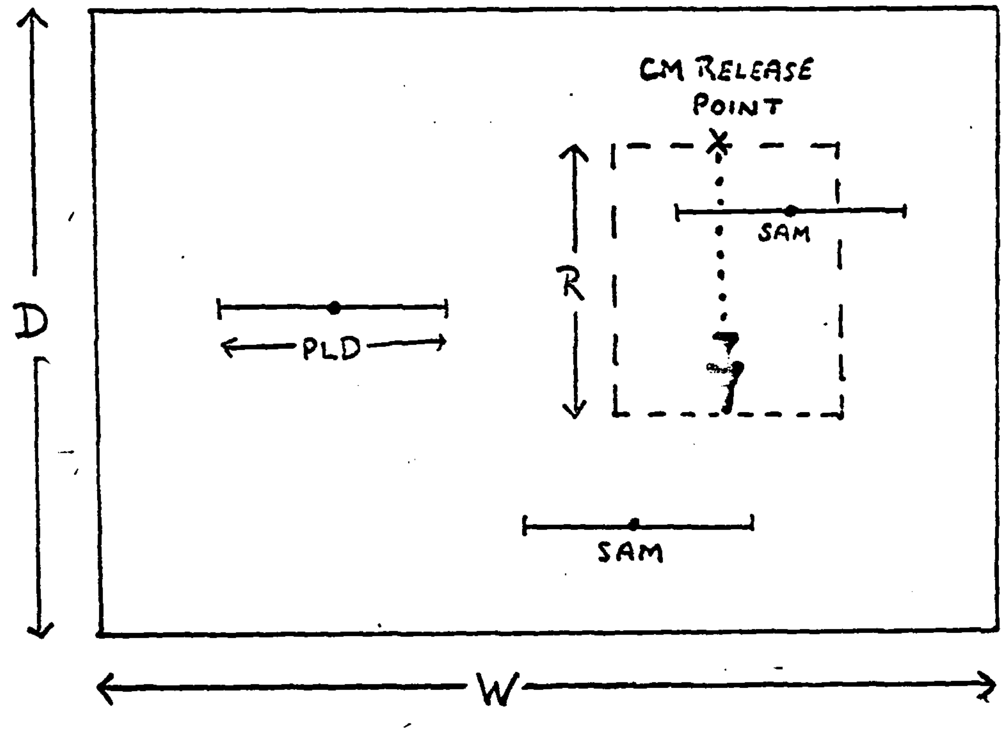

FIGURE 6.5

Cruise missile vs. random area SAMs.

detection is distributed exponentially and therefore will not depend on where the CM begins its flight. The CMs probability of penetrating a distance R through FI defenses will not depend on where it begins penetration and can be found using the analysis for FI defenses in Chapter V.

$$P_{rc}(R) = \left(\frac{P_{kfc}\lambda}{P_{kfc}\lambda-\alpha_c}\right)e^{-\alpha_c R/v_c} - \left(\frac{\alpha_c}{P_{kfc}\lambda-\alpha_c}\right)e^{-P_{kfc}\lambda R/v_c} \qquad 6.20$$

Combining equations 6.19 and 6.20, one finds the CM's probability of penetrating a distance R through the area defenses, given that it is undetected at the time it is released, to be

$$P_{Pc}(R|U) = [1-P_{ksc}\frac{w_c R}{WD}]^{N_R}[(\frac{P_{kfc}\lambda}{P_{kfc}\lambda-\alpha_c})e^{-\alpha_c R/v_c}$$

$$-(\frac{\alpha_c}{P_{kfc}\lambda-\alpha_c})e^{-P_{kfc}\lambda R/v_c}] \qquad 6.21$$

The other possible situation at the time the bomber releases the CM is that the bomber has already been detected but has still survived to reach the release point. The probability of this event, $P_x(V)$, is given by the first term on the RHS of Equation 6.16

$$P_x(v) = [1-P_{ksb}\frac{w_b x}{WD}]^{N_R}(\frac{\lambda_c}{P_{ksb}\lambda-\alpha_c})(e^{-\alpha_b x/v_b}-e^{-P_{kfb}\lambda x/v_b}) \qquad 6.22$$

If the bomber is being tracked at the time it releases the CM, it seems reasonable to assume that the CM will also be detected at the time of its release. This will not affect the CM's probability of surviving to travel a distance R through RASAM defenses. This will remain the same as given in equation 6.19. However, if the CM is detected as it is released, then the defense can immediately begin assigning interceptors to it. The total number of intercepts that can occur in the time it takes the CM to travel a distance R will be, by the assumptions made in Chapter V about FI defenses, a Poisson random variable with parameter λ. From equation 5.16, $P_i(R)$, the probability of i intercepts in a distance R is

$$P_i(R) = (\lambda R/v_c)^i e^{-\lambda R/v_c}/i! \qquad 6.23$$

To travel the total distance R through the FI defenses, the CM must survive all such attacks, each of which may kill the CM with

probability P_{kfc}. By conditioning on the number of intercepts possible and using the identity given in equation 5.18, one finds the CM's probability of surviving a distance R through the FI defenses to be

$$P_{Fc}(R) = \sum_{i=0}^{\infty} (1-P_{kfc})^i (\lambda R/v_c)^i e^{-\lambda R/v_c}/i!$$

$$= e^{-P_{kfc}\lambda R/v_c} \qquad 5.24$$

Combining equations 6.19 and 6.24 gives the probability, $P_{Pc}(R|V)$ that a CM penetrates a distance R on its own through the area defenses, given that it is detected as soon as it is released, to be

$$P_{Pc}(R|V) = [1-P_{ksc}\frac{w_c R}{WD}]^{N_R} e^{-P_{kfc}\lambda R/v_c} \qquad 6.25$$

Using the above analysis, one can now find the probability that a CM is released at a depth x and penetrates on its own an additional distance R, thus reaching its target. By conditioning this probability, denoted $P'_{CM}(x)$, on whether the bomber has reached the CM release point x, detected (V) or undetected (U), one finds

$$P'_{CM}(x) - P_{Pc}(R|V)P_x(V) + P_{Pc}(R|U)P_x(U) \qquad 6.26$$

Equations 6.19, 6.20, 6.22, and 6.25 can be substituted for the quantities on the RHS of Equation 6.26, giving, after some rearranging,

$$P'_{CM}(x) = [1-P_{ksb}\frac{w_b x}{WD}]^{N_R} [1-P_{ksc}\frac{w_c R}{WD}]^{N_R}$$

$$\{\frac{\alpha_c}{P_{kfc}\lambda-\alpha_c}\} e^{-P_{ksc}\lambda R/v_c} (e^{-\alpha_b x/v_b} - e^{-P_{kfb}\lambda x/v_b})$$

$$+ e^{-\alpha_b x/v_b}[(\frac{P_{fkc}\lambda}{P_{kfc}\lambda-\alpha_c})e^{-\alpha_c R/v_c} - (\frac{\alpha_c}{P_{kfc}\lambda-\alpha_c})e^{-P_{kfc}\lambda R/v_c}]\} \quad 6.27$$

Equation 6.27 gives the probability that a SRCM reaches its target through the area defenses as a function of its release point, x. When the CM reaches its target, it will encounter one last defense, namely a TSAM. The probability that the CM survives the TSAM will be denoted by P_{Tc}.

Equation 6.27 also fails to account for the possibility that a CM doesn't reach its target because of some kind of malfunction. Another possibility is terrain clobber. Since CMs may fly at very low altitudes, there is a chance that they would run into a natural or manmade object that rose sharply above the surrounding terrain. These non-defense related obstacles to the CM will be represented by q_c, where q_c is the probability that a CM fails to reach and destroy its target for reasons not related to the defenses.

All this can now be put together to give the probability that a SRCM with a target at x+R will be released by its bomber and will reach and destroy its target. This probability is a function of x and is denoted $P_{CM}(x)$.

$$P_{CM}(x) = P'_{CM}(x)P_{Tc}(1-q_c) \quad 6.28$$

To find an average probability that a CM reaches and destroys its target, it is necessary to know something about the distribution of the targets within the defended area. Consider a pdf for the location

(in terms of penetration depth) of a random target, $p_c(y)$. Since these targets are all assumed to be at least a distance R inside the defended area, the pdf might look something like the one shown in Figure 6.6.

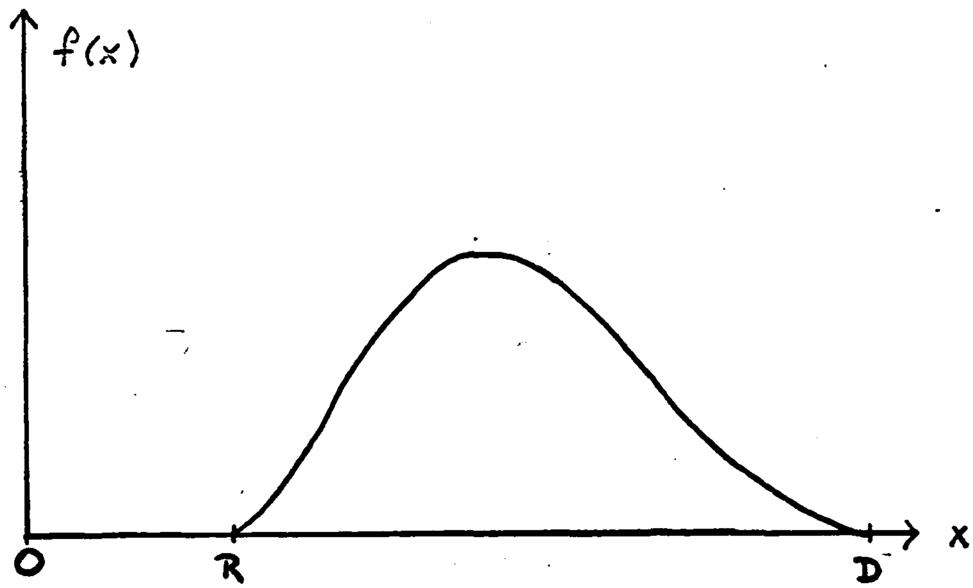

FIGURE 6.6

Density function for CM targets as a function of depth.

The probability that a CM reaches and destroys its target given in equation 6.28 is a function of its release point x, not the depth of the target y. However, note that

$$x = y - R. \qquad 6.29$$

This is because it is assumed that the CM will always be released at its maximum range R away from its target.

The average probability that a CM reaches and destroys its target, denoted P_{CM}, can now be found to be conditioning on the location of its target

$$P_{CM} = \int_R^D P_{CM}(y-R)p_c(y)dy \quad . \qquad 6.30$$

In practice, this integral would be difficult to impossible to evaluate analytically. However, a good approximation could always be found using any of several different computer integration techniques.

Once P_{CM} has been found, the distribution of the number of targets destroyed will be binomial

$$P_{dc}(k) = \binom{BC}{k} P_{CM}^k (1-P_{CM})^{BC-k} \qquad 6.31$$

where B is the number of bombers that have survived the BSAM defense and C is the number of CMs on each bomber. As in the case of weapons delivered discussed in the previous section, the use of the binomial distribution implies the assumption that each CM's probability of success is independent of what has happened to all the others. The comments made on this assumption in the section on weapons delivered are equally appropriate here.

Intermediate Range Cruise Missiles. Consider next a CM that is released after the FAD zone but before the bomber reaches the BSAM band. The CM is destined for a target at some distance y inside the defended area.

To reach its target, the CM must first survive the BSAM defense and then penetrate on its own to a depth y inside the defended area. Each part of the CM's problem can be treated exactly the same as for a bomber.

In Chapter IV, the probability that a particular bomber survived the BSAM defense was derived (Equation 4.22). For a cruise missile, the analysis procedes in the same way and one finds

$$P_{BSc} = [1-(wc/W)P_{ksc}]^{N_B} \quad . \qquad 6.32$$

If the CM survives the BSAM defense, then, like a bomber, it will begin penetration undetected. Its probability of penetrating to a depth y within the defended area is given by the penetration probability function $P_{Pc}(y)$

$$P_{Pc}(y) = [1-P_{ksc}\frac{w_c y}{WD}]^{N_R}[(\frac{P_{kfc}\lambda}{P_{kfc}\lambda-\alpha_c})e^{-\alpha_c y/v_c}$$
$$- (\frac{a_c}{P_{kfc}\lambda-\alpha_c})e^{-P_{kfc}\lambda y/v_c}] \qquad 6.33$$

The CM's total probability of reaching its target through the defenses, $P_{CM}(y)$, can now be expressed as the product of its probability of surviving the BSAM defense and its probability of penetrating the area defenses to a depth y within the country.

$$P'_{CM}(y) = P_{BSc}P_{Pc}(y)$$

$$= [1-P_{ksc}(w_c/W)]^{N_B}[1-P_{ksc}\frac{w_c y}{WD}]^{N_R}[(P_{kfc}\lambda-\alpha_c)e^{-\alpha_c y/v_c}$$

$$- \left(\frac{\alpha_c}{P_{kfc}\lambda - \alpha_c}\right) e^{-P_{kfc}\lambda y/v_c}] \qquad 6.34$$

As already described, the CM has a probability of failing to reach or destroy its target that is unrelated to the defense, q_c. And if it reaches its target, it still faces the TSAM defense (P_{Tc}). These factors combine as in equation 6.29 to give

$$P_{CM}(y) = P'_{CM}(y)\ P_{Tc}\ (1-q_c) \qquad 6.35$$

where $P_{CM}(y)$ is the probability a CM reaches its target if the target is at a depth y.

If the distribution of the CM's targets is known, the average probability that an IRCM reaches and destroys its target can be found by conditioning on the target's location.

$$P_{CM} = \int_0^D P_{CM}(y) p_c(y) dy \qquad 6.36$$

Notice that since the CMs are released outside the defended area anyway, it is no longer assumed that the targets are at a distance greater than the range of the CM inside the defended area.

Given P_{CM}, the distribution of the number of CMs that reach and destroy targets is given by equation 6.31 where B_F is the total number of bombers that survive the FAD and C is the number of CMs on each bomber.

$$P_{dc}(k|B_F) = \binom{B_F C}{k} P_{CM}^k (1-P_{CM})^{B_F C-k} \qquad 6.37$$

Long Range Cruise Missiles. Finally, consider the case of a cruise missile of sufficiently long range that it can be released before the bomber reaches the FAD zone. The CM is destined for a target at depth y inside the defended area.

To reach its target, the CM must now survive both the FAD zone and the BSAM and then penetrate to its target at a distance y inside the defended area. The analysis for the BSAM defense and for penetration against the area defenses will be the same as for IRCMs. The only thing that has to be added is the probability that the CM survives the FAD.

As in the case of both the BSAM and area defenses, the analysis of a CM's probability of surviving the FAD will exactly parallel the analysis done for bombers. For a given number n_c of fighters assigned to intercept the CM, the probability that the CM survives all of the attacks and so survives the FAD is given by Equation 4.17,

$$P_c = (1 - P_{kc})^{n_c} \qquad 6.38$$

where P_{kc} is the probability that the CM is killed by a single FAD intercepter assigned assigned to it.

It remains to determine the number of fighters, n_c, assigned to attack the CM. In the section in Chapter IV on bombers against the FAD, it was assumed that the defense would try to assign fighters uniformly to all approaching bombers. In the present case where LRCMs are added to the attack, the assumption will be broadened to assign the fighters uniformly to all approaching penetrators, including both bombers and CMs. This assumption might imply that the defense feels

that individual bombers and CMs pose a roughly equal threat or simply that the defense cannot distinguish between a bomber and a CM on radar and so assigns fighters uniformly to all penetrators.

If there are B_{BE} bombers and C cruise missiles entering a FAD zone defended by F fighters, then each penetrator will have at least n_c fighters assigned to it where

$$n_c = < F/B_{BE} + C)> \qquad 6.39$$

If $F/(B_{BE}+C)$ is a noninteger, the model assigns the remaining fighters one each to the bombers penetrating at high altitude, consistent with the analysis in Chapter IV. If there are still fighters remaining, it is arbitrarily assumed that they are assigned next to the low altitude bombers and finally to the CMs.

Equations 6.34 and 6.38 can now be combined to give the probability that a CM survives the FAD and BSAM defenses and penetrates the area defenses to its target at a depth y inside the defended area.

$$P'_{CM}(y) = (1-P_{kc})^{n_c}[1-P_{kfc}\frac{w_c}{W}]^{N_B}[1-P_{ksc}\frac{w_c y}{WD}]^{N_R}[(\frac{P_{kfc}\lambda}{P_{kfc}\lambda-\alpha_c})e^{-\alpha_c y/v_c}$$
$$- (\frac{\alpha_c}{P_{kfc}\lambda-\alpha_c})e^{-P_{kfc}\lambda y/v_c}] \qquad 6.40$$

Once again, account must be made of the CM's probability of surviving the TSAM defense and of the probability that it fails to reach or destroy its target for reasons unrelated to the defenses. This gives

$$P_{CM}(y) = P'_{CM}(y)P_{Tc}(1-q_c) \qquad 6.41$$

As before, this equation can be multiplied by the pdf of the location of the target and integrated to give

$$P_{CM} = \int_0^D P_{CM}(y)p_c(y)dy \qquad 6.42$$

Again, given P_{CM}, the distribution of the number of targets destroyed is binomial and is given in Equation 6.30 or 6.36. B_{BE} is the number of bombers that survive the base escape.

$$P_{dc}(k|B_{BE}) = \binom{B_{BE}C}{k} P_{CM}^k (1-P_{CM})^{B_{BE}C-k} \qquad 6.43$$

High vs. Low Altitude. As has been the case in all parts of this model since aerial refueling, it is necessary to consider the effect due to two groups of bombers flying at different altitudes.

It is assumed that all CMs, regardless of what altitude they are released at, will penetrate at some low altitude. With this assumption, the altitudes of the releasing bombers will not effect the distribution of the number of LRCMs that reach their targets. All the bombers that survive the base escape will release their CMs whether they are refueled or not and the CMs will all penetrate at the same altitude; thus they will all have the same probability of reaching and destroying their targets.

In the case of IRCMs, all the CMs, regardless of the altitude at which they are released, will go to low altitude and thus all have equal probabilities of reaching their targets. However, the total number of CMs released will depend on the total number of bombers that survive the FAD which will depend on the numbers of bombers penetrating at high and low altitudes.

In the case of SRCMs, each CM will have the same probability of reaching its target given that it is released, since they will all fly at the same altitudes. However, a CM's probability of being released depends on its bomber's penetration probability function which in turn depends on the altitude at which the bomber is penetrating. Therefore, in this case, two separate distributions are used, giving the number of targets destroyed by CMs released by the two different groups of bombers. These distributions can be combined to give the total number of targets destroyed in the case of SRCMs.

CHAPTER VII

INTEGRATION AND COMPUTERIZATION

The preceding chapters have described the model representations for each of the stages of the strategic bomber mission. This chapter will briefly describe how the various stages of the model are integrated for computerization.

In the program written for this paper, there are six basic outputs desired. These are a) a mean and a variance for the total number of weapons delivered, b) a mean and a variance for the total number of bombers that survive the mission, and c) a mean and a variance for the total number of cruise missiles that reach and destroy their targets. These are not the only possible outputs that might be obtained from the model; with relatively minor modifications, the program could give intermediate results such as the number of bombers that survive any particular layer of defense, for example BSAMs. The program could also be expanded to give plots of the various distributions instead of just means and variances.

Conditioning

To find the mean and variance of the quantities listed above, it is generally necessary to know their distributions. Since the number of weapons, bombers, and cruise missiles will always be integers, the distributors will all be discrete. Given a discrete distribution $P(K=k)$, the mean, $E(K)$, and variance, $Var(K)$, are expressed by the following formulas:

$$E(K) = \sum_{i} iP(K=i) \qquad 7.1$$

$$Var(K) = \sum_i i^2 P(K=i) - E(K)^2 \qquad 7.2$$

Thus, to obtain the six desired outputs, it is necessary to find three distributions, one for the total number of weapons delivered, another for the total number of bombers that survive the mission, and the third for the total number of cruise missiles that reach and destroy their targets.

Obtaining any one of these distributions is a problem in conditional probability. The reasoning process for all three distributions will be the same. Because it is convenient to deal with a specific example, the distribution for the total number of bombers that survive the mission will be used.

It is desired to find the probability that K bombers survive the mission where the initial number of bombers that begin the mission is a input to the problem. To find this probability, work backwards from the last stage of the mission, recovery, to the first, base escape.

Recall that the model is dealing with two groups of bombers, one flying at high altitude and the other flying at low altitude. Let K be the total number of bombers that survive and let K_H and K_L be the numbers that survive at high and low altitudes respectively. Then one has

$$P(K=k) = P(K_H+K_L=k) \qquad 7.3$$

By conditioning on the number of bombers that survive at high altitude, equation 7.3 can be expressed as

$$P(K=k) = \sum_h P(K_H+K_L=k|K_H=h)P(K_H=h)$$

$$= \sum_h P(K_L=k-h)P(K_H=h) \qquad 7.4$$

Now let B_{Ph} and $B_{P\ell}$ represent the number of bombers that have survived the BSAMs at each altitude. Then the two probabilities $P(K_L=k-h)$ and $P(K_H=h)$ can be conditioned on $B_{P\ell}$ and B_{Ph} respectively to give

$$P(K_H=h) = \sum_i P(K_H=h|B_{Ph}=i)P(B_{Ph}=i) \qquad 7.5a$$

$$P(K_L=\ell) = \sum_i P(K_L=\ell|B_{P\ell}=i)P(B_{P\ell}=i) \qquad 7.5b$$

where for convenience k-h has been replaced by ℓ.

An expression for the quantities $P(K_H=h|B_{ph}=i)$ and $P(K_L=\ell|B_{P\ell}=i)$ was derived in Chapter VI in the section on recovery and is given in equation 6.14.

The terms $P(B_{Ph}=i)$ and $P(B_{P\ell}=i)$ can each be expanded by conditioning on the numbers of bombers that survive the FAD at high and low altitudes respectively. Denote these numbers B_{Fh} and $B_{F\ell}$. Then

$$P(B_{Ph}=i) = \sum_j P(B_{Ph}=i|B_{Fh}=j)P(B_{Fh}=j) \qquad 7.6a$$

$$P(B_{P\ell}=i) = \sum_j P(B_{P\ell}=i|B_{F\ell}=j)P(B_{F\ell}=j) \qquad 7.6b$$

Expressions for $P(B_{Ph}=i|B_{Fh}=j)$ and $P(B_{P\ell}=i|B_{F\ell}=j)$ were derived in Chapter IV in the section on BSAM defense. They are given in Equations 4.35a and b.

The terms $P(B_{Fh}=j)$ and $P(B_{F\ell}=j)$ can each be conditioned on the numbers of bombers entering the FAD zone at each altitude. If these

numbers are denoted B_h and B_ℓ, then

$$P(B_{Fh}=j) = \sum_m P(B_{Fh}=j|B_h=m)P(B_h=m) \qquad 7.7a$$

$$P(B_{F\ell}=j) = \sum_n P(B_{F\ell}=j|B_\ell=n)P(B_\ell=n) \qquad 7.7b$$

In Chapter IV, an expression was found for the terms $P(B_{Fh}=j|B_h=m)$ and $P(B_{F\ell}=j|B_\ell=n)$. This is given by Equation 4.28.

The numbers of bombers flying at high and low altitudes is determined by the number of bombers that successfully refuel. Specifically, one has

$$P(B_h=m) = P(m \text{ bombers fail to refuel}) \qquad \text{and} \qquad 7.8a$$

$$P(B_\ell=n) = P(n \text{ bombers successfully refuel}) \quad . \qquad 7.8b$$

Each of these probabilities can be conditioned on the number of bombers that survive the base escape, B_{BE}.

$$P(B_h=m) = \sum_b P(B_h=m|B_{BE}=b)P(B_{BE}=b) \qquad 7.9a$$

$$P(B_\ell=n) = \sum_b P(B_\ell=n|B_{BE}=b)P(B_{BE}=b) \qquad 7.9b$$

Now consider that the sum of the numbers of bombers flying at high altitude and at low altitude must be the total number of bombers that survived the base escape, B_{BE}.

$$B_h + B_\ell = B_{BE} \qquad 7.10$$

Using 7.10, the conditional probability $P(B_h=m|B_{BE}=b)$ can be expressed in terms of B_ℓ as follows

$$P(B_h=m|B_{BE}=b) = P(B_\ell=b-m|B_{BE}=b) \qquad 7.11$$

Equation 7.11 can be substituted into 7.9a to give (with equation 7.9b unchanged)

$$P(B_h=m) = \sum_b P(B_\ell=b-m|B_{BE}=b)P(B_{BE}=b) \qquad 7.12a$$

$$P(B_\ell=n) = \sum_b P(B_\ell=n|B_{BE}=b)P(B_{BE}=b) \qquad 7.12b$$

The quantity $P(B_\ell=i|B_{BE}=b)$ has been derived in Chapter IV in the section on refueling. It is given in either Equation 4.18 or 4.21 as appropriate. The expression for $P(B_{BE}=b)$ was derived on the base escape section of Chapter IV and is given in equation 4.16 when the initial numbers of bombers on the gound and in the air prior to the attack are given.

This completes the integration of the model to find the distribution of the number of bombers that survive the mission. Parallel derivations can be done to give the distributions of the numbers of weaons and cruise missiles delivered on targets.

COMPUTERIZATION

In writing the computer program for this paper, the intent was to demonstrate the validity and practicality of the model that has been developed. The resulting computer model is therefore not intended or expected to be optimum in terms of flexibility or efficiency. However, it is hoped that the program might provide a starting point for someone who desired to write his own.

For convenience, one assumption and one approximation have been built into the program. These are described below.

The assumption involves the base escape part of the model. Recall that when this part was developed in Chapter IV, the distribution for the time T_A which the bombers have to take off in was left as an unspecified input. For convenience, in the computer program this distribution has been built in as a Beta distribution with parameters $\alpha=1$ and $\beta=5$. The Beta distribution is given by

$$f(x) = \frac{(\alpha+\beta+1)!}{\alpha!\beta!} x^{\alpha}(1-x)^{\beta} \qquad 0\leq x\leq 1 \qquad 7.13$$

when α and β are integers. In the model, the maximum and minimum possible values of T_A are input and the computer fits the distribution between them using the substitution $x= (t-T_{min})/(T_{max}-T_{min})$.

For illustrative purposes, this program contains the code for modelling long range cruise missiles. The approximation made in the program involves both the CMs and the bombers in the FAD part of the mission. Recall that because the numbers of fighters, bombers and CMs are all integers, it is possible to have some bombers with one more interceptor assigned to them than the other bombers or some CMs with one more inteceptor assigned to them than the other CMs. In other words, within any one of three distinct groups - high altitude bombers, low altitude bombers, or CMs, - it will be possible to have two subgroups of penetrators having different probabilities of surviving the FAD. The interceptor assignment policy which assigns remaining interceptors first to high altitude bombers, then to low altitude bombers,

and finally to CMs, prevents this subgrouping form from occurring in more than one of the three groups of penetrators.

The model treats these subgroups separately and then convolutes the distributions of the number of survivors from each subgroup to find the total number of survivors from the particular group of penetrators. This is done in equations 4.27 and 4.28. These equations are repeated here with the assumption, for the sake of example, that it is the CMs that are divided into subgroups.

$$p_{s1} = (1-p_k) \langle F/B+C \rangle + 1 \qquad 7.14a$$

$$p_{s2} = (1-p_k) \langle F/B+C \rangle \qquad 7.14\ b$$

$$P(k \text{ survive FAD}) = \sum_{i=\max(o,k-C+R)}^{\max(k,R)} \binom{C-R}{k-i} p_{s2}^{k-i} (1-p_{s2})^{C-R-k+i} \cdot \binom{R}{i} p_{s1}^{i} (1-p_{s1})^{R-i} \qquad 7.15$$

where F is the total number of fighters, B is the total number of bombers, c is the total number of CMs, and R is the number of CMs with one extra fighter assigned to them.

Since the FAD routine is called numerous times by the main computer program, time can be saved by eliminating the summation in equation 7.15. To do this, the approximation is made that all CMs have the same probability of survival which is given by the average for the survival probabilities of all the CM's

$$p_s = \frac{R}{C} p_{s1} + \frac{C-R}{C} p_{s2}$$

Using this, the probability that k CMs survive the FAD is now given by

$$p(k \text{ survive FAD}) = \binom{C}{k} p_s^k (1-p_s)^{C-k} \qquad 7.16$$

Some care should be exercised using this approximation when the number of fighter-interceptors is small compared to the number of penetrators. Since this program was written largely to demonstrate the validity and practicality of the model, the approximation is used for both bombers and cruise missiles to save computer time.

A flow chart of the program is shown in Figure 7.1. The program will not be described in detail in this paper. A listing of the program and the inputs to it is given in Appendix A. Some verification and results of the program are given in Chapter VIII.

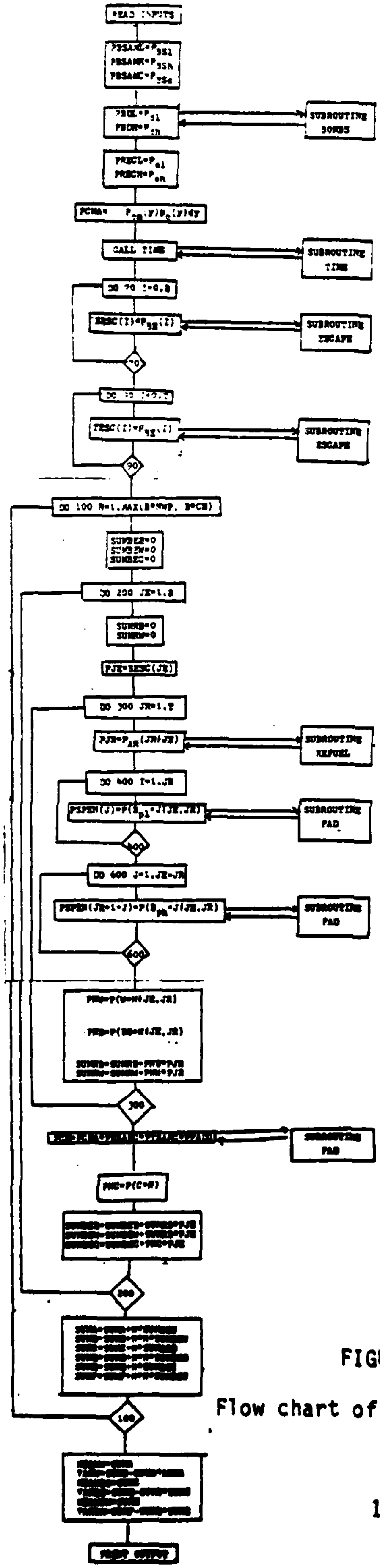

FIGURE 7.1

Flow chart of computer program.

CHAPTER VIII

VERIFICATION AND RESULTS

This chapter will briefly outline some of the steps taken to verify the computer model. Also in this chapter some initial results from the model are given.

VERIFICATION

Any computer model of this size and complexity requires careful verification to insure that the program actually does what the analyst wants it to do. Due to the overall time constraint placed on this project and various problems that arose with the computer system that was used the program may not have received the extensive verification it deserves. However, some initial steps have been taken to verify the program.

The first step taken to verify the program was to run the program for an extremely scaled down version of the problem. This same problem was also done manually and the results were compared to those from the computer version. Specifically, the problem involved the following basic inputs:

2 bombers, initially both on the ground, each armed with one weapon;

2 tankers, both initially airborne, and a bomber probability of successfully refueling given a tanker of 1;

2 FAD interceptors with a combined probability of detecting, converting on, and killing each bomber of 0.5;

1 BSAM site with a PLD of half the width of the barrier (PLD=1) and a p_k of 1 on bombers within the PLD; and

0 RASAM sites.

The intercept intensity parameter λ was set equal to 1 with area FIs having a p_k of 1. The detection parameter α was set equal to 0.5. The velocity of a bomber was 1, the size of the defended area was 2 by 2 and the bombers targets were located at a depth 1 inside the defended area. The other parameters of the model were chosen to give the bombers a probability of 0.5 of successfully escaping their base. Note that the bombers have a probability 1 of successfully refueling in this problem.

The computer program and the hand calculations gave the same results for the mean and variances of the number of weapons delivered and the number of bombers that survived the mission. At the time this initial verification was done, the code for cruise missiles was not yet contained in the program.

It should be noted that the program contains several different IF/THEN/ELSE blocks. In any simple run, such as they are described above, many small parts of the program may not be tested. To test every part of the program this way, it would be necessary to select values of the inputs that will insure that the parts of the program in question are called. The time constraints placed on this project precluded such in depth verification.

The second major form of verification undertaken for this program was to vary some of the major inputs of the model and verify that the changes in the outputs were what would be predicted.

The first parameter to be varied was the number of bombers in the problem. In general, as the number of bombers is increased, all other parameters remaining the same, the numbers of weapons delivered

and bombers surviving both increase. However, when all the bombers are initially on the ground, there is an upper limit to this increase due to the combination of the rate at which bombers can take off and the time in which they can take off before the SLBMs arrive at their bases. For example, if the missile flight time is twelve minutes and bombers take off at a rate of one per minute, there is nothing to be gained by increasing the number of bombers initially on the ground beyond twelve.

The numbers of defenses have also been varied with the program yielding decreases in weapons delivered and bombers surviving as the numbers of defenses, such as FAD intercepters and SAMs, are increased. The same holds true when the intercept intensity paramter λ is increased, which is the same as increasing the level of area FI defenses in the problem.

The last major category of parameters to be varied included the probabilities of detection and conversion and the kill probabilities for the different defenses. In general, two results are expected. First, as the probabilities are increased, corresponding to improvements in defensive capabilities, the numbers of weapons delivered and bombers surviving should both decrease. Second, it was to be expected that probabilities near 0.5 will give the largest variances for the results while probabilities near 0.1 and 0.9 will give smaller variances. Both of these expectations are born out by the program.

Because the code for the cruise missiles was added after the other code had been written, verification of the outputs for CMs was less extensive than for the weapons delivered and bombers surviving.

However, there are at least two indications that the CM part of the program works as it should. First, as the number of CMs in the problem is increased, the expected number that reach and destroy targets also increases. Second as the number of CMs are increased the expected numbers of weapons delivered and bombers surviving also initially increases. Since the program is written for long range CMs, increases in the number of CMs increase the number of penetraters entering the FAD zone, initially giving each one a better chance of surviving. After the number of CMs reaches a certain level, there will be only one intercepter assigned to each bomber and all the rest of the intercepters will be assigned to CMs. Further increases in the number of CMs will not improve the number of bombers surviving or weapons delivered.

RESULTS

The following is a brief overview of some of the results obtained from the model. These results are intended to show that the model works and give some feel for the output that can be obtained. The inputs have been selected solely for this purpose and are not necessarily meant to describe any specific realistic situation.

The base case for all of the results to be presented below is as follows:

15 bombers, initially all on the ground, each carrying 3 weapons;

15 tankers, initially all on the ground;

30 FAD fighter-intercepters;

50 BSAMs; and

100 RASAMs.

The intercept intensity paramter λ was set to 0.5. The size of the defended area was 4000 by 4000. SAM sites had a PLD of 3. A bomber's probability of refueling, given a tanker was set to 0.95. All other probabilities were set to 0.5. The detection parameter α was set to 0.5 for low altitude bombers and 1.0 for high altitude bombers. This was the only distinction made between the two groups of bombers.

To reduce the amount of data to be displayed, and because the cruise missile code had received less verification than that for bombers and weapons released, cruise missiles were left out of the problem.

The results of the test case were as follows:

Expected number of weapons released - 2.31

Variance of number of weapons released - 9.10

Expected number of bombers surviving - 0.29

Variance of number of bombers surviving - 0.33

Four different parametric variations were done on the test case. These are described below.

In the first example, the number of weapons on each bomber was varied between 1 and 5. The results of the expected number of weapons delivered and the expected number of bombers that survive the mission are shown in Figure 8.1. Two things can be noted. First, the expected number of bombers that survive the mission decreases as the number of weapons on each bomber increases. This is because the bombers must face terminal SAM defenses at each target they attack. Also, as the number of weapons on each bomber increases, the rate of increase of the expected number of weapons delivered decreases. This is also a result

of bombers having to face more terminal SAM defenses; the bomber has a much smaller probability of surviving to release all its weapons.

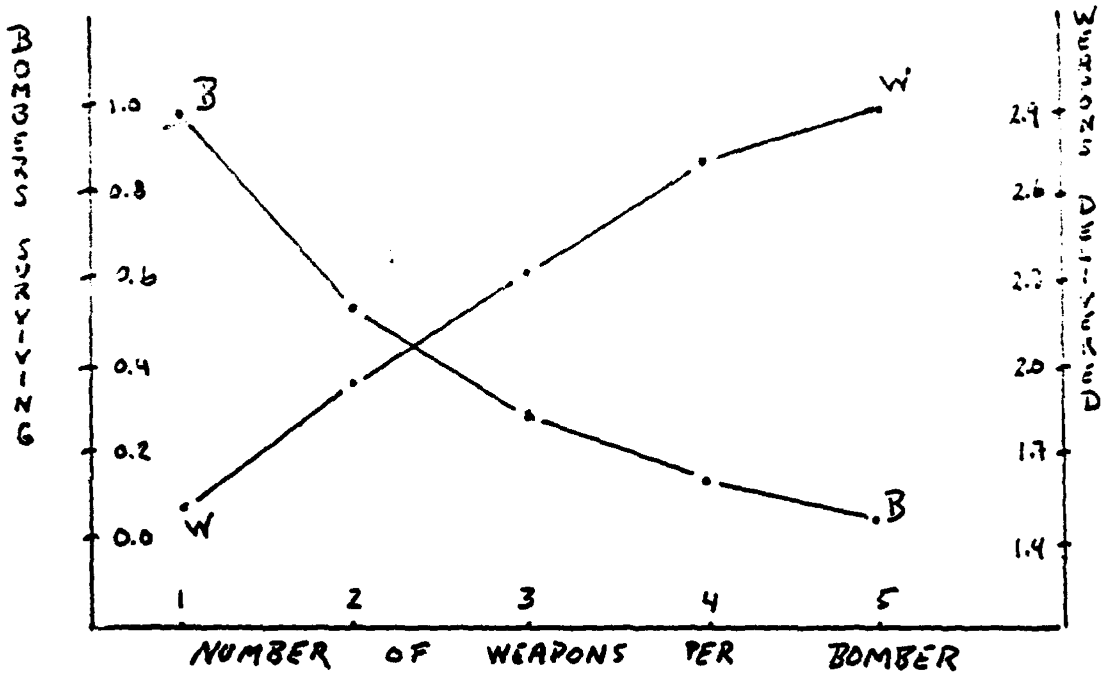

FIGURE 8.1

Bombers surviving and weapons delivered as a function of the number of weapons on each bomber.

In the second example, the bombers' probability of surviving the TSAM defenses at each target was varied from 0.1 to 0.9. The results are shown in Figure 8.2. As expected, more weapons are delivered and more bombers survive the mission as the probability of surviving the TSAMs increase.

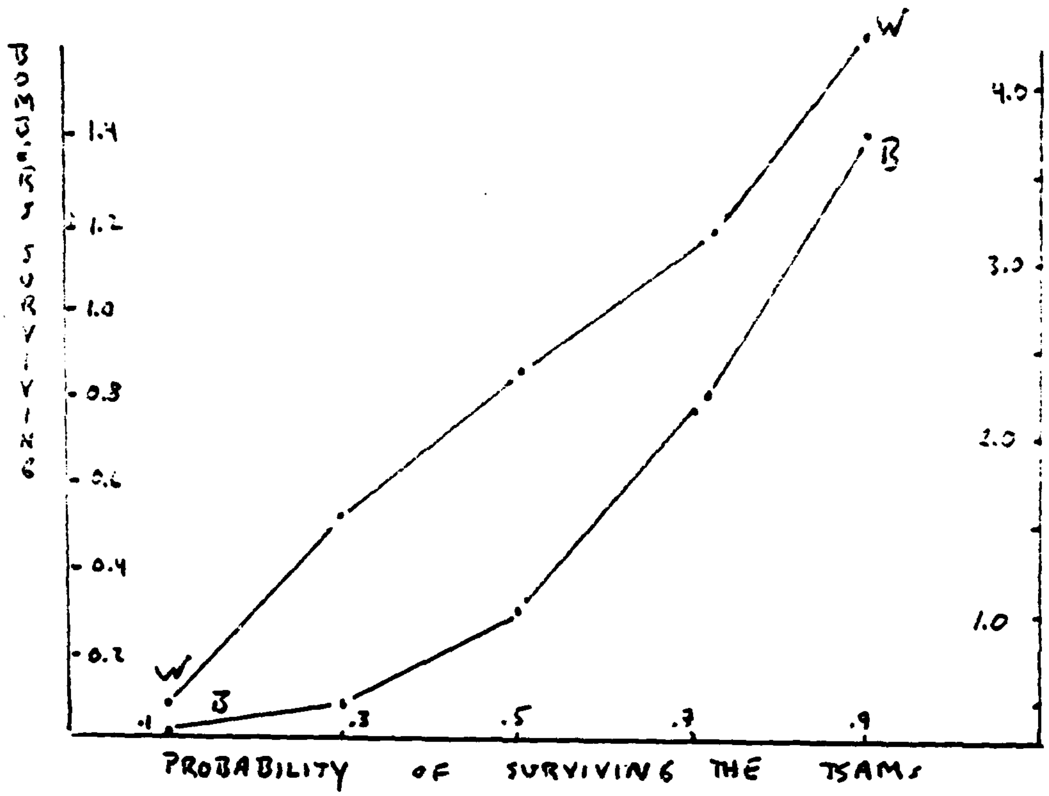

FIGURE 8.2

Bombers surviving and weapons delivered as a function

of the probability of surviving TSAM defenses.

In the third example, the intercept intensity parameter λ is varied from 0.1 to 0.9. The results are shown in Figure 8.3. As the intercept intensity increases, more FI attacks are expected on a bomber per unit time. As a result, the expected numbers of weapons delivered and bombers surviving decreases with increases in λ.

In the last example, all of the probabilities input to the model (except refueling) are varied between 0.1 and 0.9. Increases in the defensive probabilities of kill on a bomber and increases in a bombers probability of surviving the TSAM defenses tend to have offsetting effects on the expected numbers of bombers surviving and weapons released.

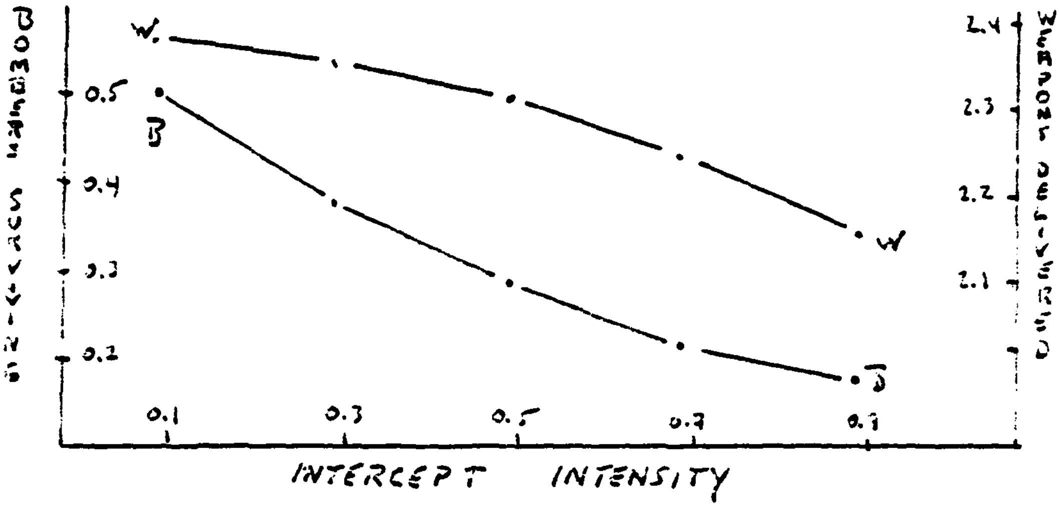

FIGURE 8.3

Bombers surviving and weapons delivered as a

function of intercept intensity.

This example was chosen to show the effects these changes will have on the variances of the model outputs. The results are shown in Figure 8.4.

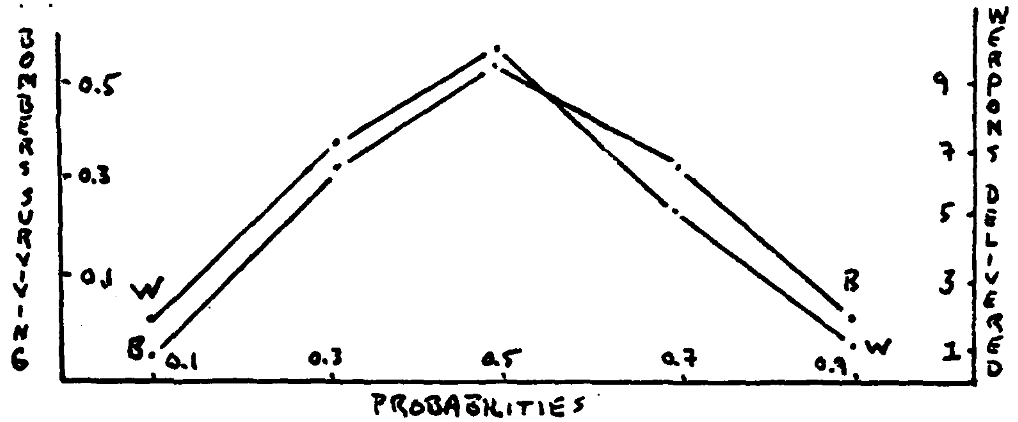

FIGURE 8.4

Variances of model outputs as a

function of the input probabilities.

This has just been a small sample of the various results that can be obtained with this model. However, they should be sufficient to indicate that the model works and to demonstrate some of its potential power.

CHAPTER IX

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this chapter is to recap the model that has been presented in this paper and to suggest some possible areas where further work could produce significant improvment or extensions to the model. At the end of this chapter, some comments and conclusions about this project will be made.

SUMMARY

This paper set out to develop an analytic model of strategic bomber penetration from which both expected value and variance calculations on the results could be made. A model of the strategic bomber mission has been constructed which covers the entire mission from the time bombers take off from their bases until they leave the defended area after releasing their weapons. It was recognized from the outset that the scope of the problem would ultimately limit the depth to which each part of the mission could be modelled; however, it is felt that the current model contains sufficient detail to adequately represent the bomber mission for many purposes. A computer program has been written that demonstrates some of the various possible outputs that can be obtained from the model. In particular, the program shows that the model can give variance calculations on its results, as desired.

There probably exists no model of this scope which can claim to be perfect in all respects and the same applies to the model presented in this paper. In the following sections, several areas are suggested where additional work could yield substantial improvements to the

model. In some cases possible approaches or potential directions for work are indicated; the other areas are open to suggestions.

COMPUTERIZATION

The computer program written for this paper was designed to demonstrate the validity and practicality of this model. The program will give the six basic results described in Chapter VII but as it currently stands, it will not output any of the numerous intermediate results of the model that may be of interest to users.

In the current program, it is not easy to model only parts of the problem. For example, by inputting an initial number of tankers in the air that is greater than the total number of bombers in the problem and by setting the probability of successfully refueling given a tanker to 1, it is possible to insure that all bombers get refueled. This will effectively leave the refueling stage of the mission out of the problem as far as the final results are concerned. However, the computer will still run the refueling routine and spend the time calculating all the various probablities. This is not efficient. In short, the computer model lacks flexibility.

Therefore, for this model to achieve its full potential value, the most important next line of work is to develop a flexible, efficient, well documented program for general use.

INTERCEPT DENSITY FUNCTION

A major part of this model rests on the assumption that a time independent paramter (λ) can be found that gives the average number of intercepts possible on a penetrating bomber per unit time. In this model, this parameter has been assumed to be input and minimum

consideration has been given to the derivation or selection of a value for the parameter. This opens several possible lines of further research.

The first thing that should be done is to find a derivation for the parameter that incorporates as much information about the defensive fighter-interceptor force as possible. An iterative procedure to find a time-dependent function for the parameter has already been developed by D. H. Monahan for COPEM-I (Ref 12:3). This procedure is described in Appendix B. It should be possible by making some appropriate assumptions to modify the method to give a constant instead of time dependent parameter.

There is an obvious alternative approach to the problem. Since a derivation for a time dependent form of the parameter already exists, and since there are good reasons to believe that such a parameter would in fact be time dependent, it might be worth some study to determine if it would be possible to modify the model to allow for the use of a time dependent parameter. The major disadvantage to such an approach lies in the fact that the penetration probability function derived in Chapter V would no longer have a closed analytic solution. The computer model would have to be expanded first to calculate the parameter and second to do a numerical intergration of the penetration probability function. This problem could only be avoided if some form of analytic function could be found to describe the parameter that would still allow the integral to be calculated analytically.

If the intercept density parameter is not a function of time, one would still expect it to depend on the relative numbers of penetrators and fighters in the air-to-air battle over the defeneded area. In this paper, the parameter has been implicitly assumed to be the same for all possible numbers of penetrators.

Two possible approaches to this problem exist. First, an expected initial number of penetrators in the air-to-air battle could be used to calculate the parameter. This would most likely give offense conservative results for the total expected number of weapons delivered in the attack. However, it would also tend to understate the variance of the attack.

The second possible solution to this problem is to modify the FI part of the model to permit the parameter to be a function of the number of penetrators that survive the BSAM defense. The way the current model is written, this would not be a difficult change to make. An array giving the parameter as a function of the number of penetrators could be input, or, if an analytic approximation to the function could be found, the computer could perform the calculations during each run of the model.

WEAPONS DELIVERED

In order to keep the scope of this problem down to something that could be handled in the time frame of this paper, several assumptions were made in the sections on terminal SAMs, weapons delivered, and value extracted. These assumptions were made for modelling purposes and it is recognized that, from the point of view of realism, these are probably among the least defensible assumptions in this model.

For this reason, it is suggested that a major area for further analysis on this model is the terminal SAM and weapons delivered section. This would probably have to be combined with some form of weapon allocation scheme for the model which would in itself be a significant improvement to the model.

The Schultis treatment of TSAMs and value extracted might provide a good starting point for such work; however, it will have some limitations if one tries to apply it directly to this model. For one thing, the Schultis model gives expected value damage calculations based on saturation of the terminal defenses. This may not be an optimum strategy in the first place and if it is, it still gives no indication of the variability in the results, which was one of the major goals of this model.

Another problem that should be addressed is the relationship between the terminal SAM defenses and the depth of the location of the target inside the defended area. In the Schultis model, targets are defended strictly according to their value and their relative location is ignored. This may not be the actual case.

One last problem to consider is the uncertainty that arises because weapons are allocated to targets before the mission instead of just before the weapons reach targets. The Schultis model gives calculations to show how much damage could be extracted by a particular number of weapons that survive all the other defenses, if the weapons were free to be allocated at that point. Unfortunately, weapons must be allocated to targets before the mission begins and weapons bound for

high value targets have as much chance of being lost to early defenses as weapons destined for lower value targets. This needs to be considered when weapons are being allocated and should be incorporated in this model when calculations are being done at the mean and variance of the damage extracted by an attack

OTHER AREAS FOR STUDY

There are several other areas in which work could be done to improve this model.

Base Escape. Two possible suggestions to improve the model representation of the base escape problem are offered. One would be to change the geometry employed in the problem. This could include substituting some more likely shape for the volume within which the airborne bombers will be located. A more important change might be to overlap the lethal volumes from various warheads targeted on a base.

The second suggestion is related to the first. The model implicitly assumes that all the bombers that take off from the base before the warheads arrive have equal chances of surviving. In reality, the bombers that take off first will almost surely survive while a bomber which has just left the runway when the warheads begin to detonate will have little chance of surviving. One possible way to account for this would be to fix the volume within which the detonating warheads will probably lie and then find the number of bombers that took off but are still inside that potentially lethal volume of airspace. These would be the most recent aircraft to take off and their survival probabilities could be found as in this model. The aircraft that had taken off first would be outside the space where the warheads would detonate and so would be assured of surviving.

Refueling. The refueling part of the mission is a complex problem in logistics complicated by the uncertainties that arise when one models a pre-emptive SLBM strike against the strategic bomber and tanker forces. Very little work has been done in the past toward analytically modelling this problem. This paper dealt with the refueling problem in greater depth than most analytic models; however, there remains room for greater detail in the model.

BDMs. Bomber defense missiles were not considered in this model. At this time, that is probably not a serious drawback to the model; however, the flexibility of the model would be increased if they were added.

Decoys. Bomber decoys were not explicitly included in this model although the user can consider them when selecting a value for the intercept density parameter (See Ref 5, Appendix A). Penetrating bombers may find the use of decoys advantageous in the forward air defense stage of this mission. At least one approach to analytically modelling decoys can be found in the PENEX model (Ref 2).

Command and Control. In this model, all defensive fighter-inteceptors were assumed to be close controlled. This requires an effective defensive command and control system. This model does not consider ways on which the C^2 system's effectiveness could be degraded or the course of a mission. There are at least three possibilities.

First, false tracks may be generated on the defensive radars. Interceptors would be assigned to these false tracks, which would act and could be modelled similar to bomber decoys.

Another possibility is the failure of a C^2 system to detect a bomber. In this model, this possibility is accounted for in the battle over the heartland by the distribution of the time to detection; however, it is not considered in the forward air defense battle. The PENEX model demonstrates one approach to this problem.

The third possibility is the saturation or even destruction of some elements of the C^2 system. This is simulated in the APM model but a good analytic approach would be valuable.

Approximations. A few approximations were made for convenience on this model. It would perhaps be worth investigating them to determine under what conditions they are reasonable. In addition, improved or new approximations for some of the calculations may be desirable. This could become important if changes to the model result in large increases on the amount of computer time required to run it.

Validation. At some time in the future, perhaps after many of the changes to the model described above have been made, it might be worthwhile to compare some of the results of this model to a large simulation such as the APM. This would be an excellent way to validate the model and determine when its results provide a reasonable description of the bomber penetration mission.

CONCLUSIONS

This paper has described an analytic model of the entire bomber penetration mission. The model has been developed to allow calculations of the variance of the model results.

Although the original suggestion for this paper came for the Aeronautical Systems Division, Deputy for Strategic Systems, it is hoped that some of the advantages of this model will make it useful to others as well. The model's two biggest advantages are its scope, in terms of the entire bomber penetration mission, and its capability to calculate variances for its results.

The advantage to modelling all the stages of the bomber penetration mission is that it allows the results to reflect the impact of all of the events that could effect the outcome of the mission. For example, in a model that only describes bomber penetration of air defenses, the number of bombers that survive the base escape and successfully refuel must be input. This will not reflect the uncertainty that is inherent in the estimates of the number of bombers that escape and refuel. With this model, if one desires to use the model for just the penetration of air defenses, that can be done by excluding the base escape and refueling portions.

The advantage to knowing the variance of the results of a model is that it provides an idea of the confidence of the results. Another way of thinking of it is that the variance gives an indication of the spread of values that are being represented by the model's expected value result. This in turn will provide a better understanding of the significance of the results.

Potential users of this model will each have their own ideas about how the model most effectively could be used. However, to conclude this paper, two possible uses will be suggested.

First, this model might be used to perform excursions for studies where the APM is the primary model. Because of the APM's long run time, it is not practical to use it to perform parametric variations or sensitivity analysis. As an alternative, some of the numerous outputs from the APM could be used as inputs to this model. This model could then be used to do the parametric variations and sensitivity analysis for the APM results.

The second way in which this model might be used is in conjunction with a force on force attrition model such as the Arsenal Exchange Model. This model could provide the AEM with a more detailed description of the air breathing element of the strategic triad than is currently included.

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APPENDIX A

COMPUTER PROGRAM

This appendix give a listing of the program written for this model. Since the program itself contains no comments, a brief outline of the program is presented. In particular, the purpose of each of the functions and subroutines will be explained. In the main program, the purposes of the various DO loops will be outlined. The necessary inputs to the program are also defined. The program is written in FORTRAN 5.

INPUTS

Values of the following variables must be read in the program (see the READ statements for correct order; format is freefield).

INBA - initial number of bombers airborne;

INBG - initial number of bombers on the groud;

INTA - initial number of tankers airborne;

INTG - initial number of tankers on the ground;

VCB - bomber rate of climb during base escape;

VHB - horizontal velocity of bomber during base escape;

VCT - tanker rate of climb during base escape;

VHT - horizontal velocity of tanker during base escape;

NWP - number of weapons on each bomber;

CM - number of cruise missiles on each bomber;

RB - bomber take off rate;

RT - tanker take off rate;

M - number of SLBM warheads targeted at base;

TMAX - maximum time in which bombers can take off;

TMIN - minimum time in which bombers can take off;

RL - lethal radius of SLBM warheads;

PREF - bomber's probability of refueling given an available tanker;

F - number of FAD fighter-intercepters;

WL - PLD of SAM site;

NBSAM - number of BSAM sites;

NRSAM - number of RASAM sites;

W - width of defended area;

D- depth of defended area;

LAMBDA - intercept intensity parameter; and

VCM - velocity of a cruise missile.

In the following inputs, the last letter of each variable will be H, L, or C, corresponding to bombers at high and low altitudes and cruise missiles respecitively.

$pdcw_x$ - probability of detection and conversion for FAD intercepter;

$pdcl_x$ - probability of detection and conversion for area fighter-intercepter

pkw_x - probabiity of kill given detection and conversion for FAD intercepter;

pkl_x - probability of kill given detection and conversion for area fighter-intercepter;

$pksam_x$ - SAM probability of kill given penetrater enters SAM coverage;

$ptsam_x$ - probability of surviving TSAM defense at target;

$alpha_x$ - detection parameter; and

pw_x - probability that a weapon that is released successfully destroys its target.

The following variables correspond to target locations (see Chapter VI).

RH(I) - these are the x_i for bombers;

NCMR - number of intervals defended area has been divided into to do numerical integration over location of cruise missiles target;

LMR(I) - the possible locations (depths) of the cruise missile targets; and

PCMR(I) - probability that a cruise missile target is at depth CMR(I).

OUTPUTS

There are six outputs for the model. (This does not include some of the basic inputs which are listed out at the end of each run for convenience).

MEANW - expected number of weapons delivered;

VARW - variance of number of weapons delivered;

MEANBS - expected number of bombers that survive the mission;

VARBS - variance of number of bombers that survive the mission;

MEANCM - expected number of cruise missiles that reach their targets; and

VARCM - variance of the number of CMs that reach their targets.

INTERMEDIATE VARIABLES

Many intermediate variables are calculated in this program. Some of the more meaningful ones are listed here.

$PBSAM_X$ - probability a penetrater survives the BSAM defense;

PBD_X - probability a weapon is released over a target;

$PREC_X$ - probability a bomber survives to leave the defended area and recover; and

PCM - probability a CM reaches its target.

FUNCTIONS

There are four external functions employed in this program.

COMB(N,K) - calculates $\binom{N}{K}$;

XTOP(X,P) - finds X^P, with 0^0 defined as 1 for purposes of the binomial distribution;

FP(X) - probability density function for the Beta distribution with parameters $\alpha=5$ and $\beta=1$; and

FE(X) - used to find the expected value of the Beta distribution over a particular interval.

SUBROUTINES

There are five subroutines called in the program.

TIME. SUBROUTINE TIME takes TMAX and TMIN and fits a Beta distribution between them. It then finds the distribution for the number of aircraft (bombers or tankers) that can become airborne before the SLBM warheads at their base. It also finds the expected time the bombers had in which to take off given the number of bombers that did take off. (See Chapter IV, the section on base escape.)

ESCAPE. SUBROUTINE ESCAPE takes the distribution of the number of aircraft (bombers or tankers) that can get off the ground during an SLBM attack and finds the probability that a specified number of aircraft survive the base escape problem.

REFUEL. SUBROUTINE REFUEL takes the distribution for the number of tankers that have survived the base escape problem and finds the probability that a specified number of bombers successfully refuel for a given number of bombers that have survived the base escape.

FAD. SUBROUTINE FAD finds penetraters' probabilities of surviving the forward air defense. In the case of bombers, FAD will find the probability that a specified number of bombers at a given altitude (high or low) survive given the total number of penetraters entering the forware air defense. For cruise missiles, FAD finds the probability that any cruise missile survives the forward air defense, given the total number of penetraters.

BOMBS. SUBROUTINE BOMBS finds the probability that a weapon is released over a target.

MAIN PROGRAM

In this section, the purpose of each DO loop will be briefly explained. DO loops will be referred to by their CONTINUE statement numbers. The index variable for the loop may be included in parentheses.

5,15 - variable initialization loops;

50 - finds the probability a LRCM penetrates the area defenses to reach its target;

60,70 - find the distribution of the number of bombers, including those initially on airborne alert, that survive the base escape; and

80,90 - find the distribution for the number of tankers that survive the base escape.

100(N) - the number of weapons released, bombers surviving, and/or cruise missiles that reach targets. N will be incremented from 1

to the maximum of the total number of weapons or cruise missiles in the problem. Fore each N, the probability that N weapons are released, that N bombers survive, and that N cruise missiles reach their targets will be calculated, resulting in distributions for all three outputs.

200(JE) - conditions the distributions described above on the number of bombers (JE) that survive the base escape; and

300(JR) - conditions the distribution on the number of bombers that successfully refuel.

400,500 - for given JE and JR, a distribution for the number of low altitude bombers that survive both the FAD and BSAM defenses is found and stored in array PSPEN.

600,700 - for given JE and JR, a distribution for the number of high altitude bombers that survive the FAD and BSAMs is found and stored in the array PSPEN.

800,900,1000 - find the probability that a total of N weapons are delivered by the high and low altitude bombers. This probability is conditioned on the number of weaons released by the low altitude bombers.

100,1200,1300 - find the probability that a total of N bombers from both the high and low altitude groups survive the mission. The probability is conditioned in the number of low altitude bombers that survive.

Two last points will be made about the main program. The first concerns lines 1620, 1670 and 1740. These lines provide a time saving approximation to the program. Although most of the calculations in the program are DOUBLEPRECISION, it will frequently be the case that accuracy to one or two decimal places will be sufficient. In this

program, if the probability that a particular number of bombers survive the base escape is very small, subsequent calculations in the 200 loop will be skipped since they will not contribute any significant change to the final result. The 300 loop will be skipped if the probability that a particular number of bombers refuel is too small to contribute to the final results. The way the program is written, one decimal place accuracy can be had with up to 100 bombers in the problem. In the results presented in Chapter VIII, where only 15 bombers were modelled, the accuracy was better than two decimal places. If time is not a major consideration in the program, these lines can be deleted. As the program is improved, better approximations may be found.

The second point to be made concerns the cruise missile code. It should be noted that the probability that a CM fails to reach its target due to nondefense related reasons (q_c, see Chapter VI) has not been explicitly included, implying that it is always 1. This can be remedied by reading a QC and altering line 2300 to read:

```
2300=      PCM=PCMA*PRI*PBSAMC*PTSAMC*(1-QC)
```

This concludes the description of the computer program used in this model. This appendix is not meant to provide detailed documentation of the program. However, it is hoped that with the help of this description, the program can be understood by someone familiar with the model and, in particular, the integration of the model described in Chatper VII.

A listing of the program follows.

```
      PROGRAM MODEL
      INTEGER BT,F,H,I,INBA,INBG,INTA,INTG,J,JE,JR,K,L,M,N
      INTEGER NBSAM,NRSAM,NWP
      INTEGER C,CM,NCMR
      REAL ALPHAH,ALPHAL,D,LAMBDA,PDCLH,PDCLL,PDCWH,PDCWL
      REAL PKLH,PKLL,PKWH,PKWL,PKSAMH,PKSAML,PREF
      REAL RB,RL,RT,TMAX,TMIN,VCB,VCT,VHB,VHT,W,WL,VBH,VBL
      REAL PDCWC,PDCLC,PKWC,PKLC,PKSAMC
      REAL PWH,PWL
      REAL CMR(1:10),PCMR(1:10)
      DOUBLEPRECISION PTSAMC,PTSAMH,PTSAML
      DOUBLEPRECISION A,PNC
      DOUBLEPRECISION MEANBS,MEANW,VARBS,VARW,PBSAMH,PBSAML,PBDH,
PBDL
      DOUBLEPRECISION PH,PL,PJR,PJE,PNB,PNW,PRECH,PRECL,PRI,SUM
      DOUBLEPRECISION SUMA,SUMB,SUMC,SUMD,SUMBEB,SUMBEW,SUMRB,SUM
RW,SUMT
      DOUBLEPRECISION CA,CB,CC,PCM,PCMA
      REAL RN(0:25)
      DOUBLEPRECISION PSPEN(0:201)
      DOUBLEPRECISION BESC(0:100),TESC(0:200)
      DOUBLEPRECISION BET(0:100),BAIR(0:100),TET(0:200),TAIR(0:20
0)
      DOUBLEPRECISION FP,FE
      DOUBLEPRECISION SUME,SUMF,SUMBEC,MEANCM,VARCM,PBSAMC
      DOUBLEPRECISION X,Y,Z,COMB,XTOP
      READ*, INBA,INBG,VCB,VHB,RB,VBH,VBL,NWP
      READ*, INTA,INTG,VCT,VHT,RT,PREF
      READ*, PWH,PWL
      READ*, M,RL,TMAX,TMIN
      READ*, F,PDCLH,PDCLL,PDCWH,PDCWL,PKLH,PKLL,PKWH,PKWL
      READ*, W,D
      READ*, NBSAM,NRSAM,PKSAMH,PKSAML,PTSAMH,PTSAML,WL
      READ*, ALPHAC,ALPHAH,ALPHAL,LAMBDA
      READ*, (RN(I),I=1,NWP)
      READ*, PDCWC,PDCLC,PKWC,PKLC,PKSAMC,PTSAMC
      READ*, CM,NCMR,VCM
      READ*, (CMR(I),I=1,NCMR)
      READ*, (PCMR(I),I=1,NCMR)
      PRINT'(A)','1'
      PRINT'(/A,I6,A,F10.4,A,F10.4,A,I10)',
     :     ' INBA=',INBA,'  VCB=',VCB,'  VBL=',VBL,'  NWP=',NWP
      PRINT'(/A,I6,3(A,F10.4))',
     :     ' INBG=',INBG,'  VHB=',VHB,'  VBH=',VBH,'  RB=',RB
      PRINT'(/A,I6,A,F10.4,A,F9.4)',
     :     ' INTA=',INTA,'  VCT=',VCT,'  PREF=',PREF
      PRINT'(/A,I6,A,F10.4,A,F11.4)',
     :     ' INTG=',INTG,'  VHT=',VHT,'  RT=',RT
      PRINT'(/2(A,F10.6))',' PWH=',PWH,'  PWL=',PWL
      PRINT'(/A,I9)',' F=',F
      PRINT'(/4(A,F8.6))',
     :   ' PDCWL=',PDCWL,'  PDCLL=',PDCLL,'  PKWL=',PKWL,'  PKLL=
',PKWL
```

```
      PRINT'(/4(A,F8.6))',
     :   ' PDCWH=',PDCWH,'  PDCLH=',PDCLH,'  PKWH=',PKWH,'  PKLH=',PKLH
      PRINT'(/A,F11.6)',' WL=',WL
      PRINT'(/A,I6,3(A,F8.6))',
     :      ' NBSAM=',NBSAM,'  PKSAML=',PKSAML,
     :      '  PTSAML=',PTSAML,'  PKSAMC=',PKSAMC
      PRINT'(/A,I6,3(A,F8.6))',
     :      ' NRSAM=',NRSAM,'  PKSAMH=',PKSAMH,
     :      '  PTSAMH=',PTSAMH,'  PTSAMC=',PTSAMC
      PRINT'(/A,I9,A,F10.5)',
     :      ' M=',M,'  TMAX=',TMAX
      PRINT'(/A,F8.2,A,F10.5)',
     :      ' RL=',RL,'  TMIN=',TMIN
      PRINT'(/2(A,F10.5))',
     :      ' ALPHAL=',ALPHAL,'  ALPHAC=',ALPHAC
      PRINT'(/2(A,F10.5))',
     :      ' ALPHAH=',ALPHAH,'  LAMBDA=',LAMBDA
      PRINT'(/A,F11.4,A,F12.4)',
     :      ' W=',W,'  D=',D
      PRINT'(/A,I10,A,F10.4)',
     :      ' CM=',CM,'  VCM=',VCM
      PRINT'(/2(A,F10.4))',' PDCWC=',PDCWC,'  PKWC=',PKWC
      PRINT'(///A)',' '
      H=0
      L=1
      C=2
      MEANW=0.
      MEANBS=0.
      VARBS=0.
      VARW=0.
      MEANCM=0.
      VARCM=0.
      BT=INBA+INBG
      X=1.-PKSAML*WL/W
      PBSAML=XTOP(X,NBSAM)
      Y=1.-PKSAMH*WL/W
      PBSAMH=XTOP(Y,NBSAM)
      Z=1.-PKSAMC*WL/W
      PBSAMC=XTOP(Z,NBSAM)
      CALL BOMBS(NWP,PKSAMH,WL,W,D,RN,NRSAM,PDCLH,PKLH,LAMBDA,ALPHAH,
     :          VBH,PTSAMH,PBDH)
      CALL BOMBS(NWP,PKSAML,WL,W,D,RN,NRSAM,PDCLL,PKLL,LAMBDA,ALPHAL,
     :          VBL,PTSAML,PBDL)
      PBDH=PBDH*PWH
      PBDL=PBDL*PWL
```

```
      SUMA=(PDCLL*PKLL*LAMBDA/(PDCLL*PKLL*LAMBDA-ALPHAL))
     :       *EXP(-ALPHAL*D/VBL)
      SUMB=(ALPHAL/(PDCLL*PKLL*LAMBDA-ALPHAL))
     :       *EXP(-PDCLL*PKLL*LAMBDA*D/VBL)
      SUMC=XTOP(X,NRSAM)
      PRECL=SUMC*(SUMA-SUMB)*XTOP(PTSAML,NWP)
      SUMA=(PDCLH*PKLH*LAMBDA/(PDCLH*PKLH*LAMBDA-ALPHAH))
     :       *EXP(-ALPHAH*D/VBH)
      SUMB=(ALPHAH/(PDCLH*PKLH*LAMBDA-ALPHAH))
     :       *EXP(-PDCLH*PKLH*LAMBDA*D/VBH)
      SUMC=XTOP(Y,NRSAM)
      PRECH=SUMC*(SUMA-SUMB)*XTOP(PTSAMH,NWP)
      DO 5 I=0,100
      BAIR(I)=0.
      BET(I)=1.
5     CONTINUE
      DO 15 I=0,200
      TAIR(I)=0.
      TET(I)=1.
15    CONTINUE
      CALL TIME(TMAX,TMIN,RB,BAIR,BET,INBG)
      CALL TIME(TMAX,TMIN,RT,TAIR,TET,INTG)
      SUM=0.
      DO 50 I=1,NCMR
      CA=(PDCLC*PKLC*LAMBDA/(PDCLC*PKLC*LAMBDA-ALPHAL))
     :   *EXP(-ALPHAL*CMR(I)/VCM)
      CB=(ALPHAL/(PDCLC*PKLC*LAMBDA-ALPHAL))
     :   *EXP(-PDCLC*PKLC*LAMBDA*CMR(I)/VCM)
      Z=1.-PKSAMC*WL*CMR(I)/(W*D)
      CC=XTOP(Z,NRSAM)
      SUM=SUM+CC*(CA-CB)*PCMR(I)
50    CONTINUE
      PCMA=SUM
      DO 60 I=0,INBA-1
      BESC(I)=0.
60    CONTINUE
      DO 70 I=INBA,INBA+INBG
      K=I-INBA
      CALL ESCAPE(K,INBG,M,RL,BAIR,BET,VHB,VCB,BESC(I))
70    CONTINUE
      DO 80 I=0,INTA-1
      TESC(I)=0.
80    CONTINUE
      DO 90 I=INTA,INTA+INTG
      K=I-INTA
      CALL ESCAPE(K,INTG,M,RL,TAIR,TET,VHT,VCT,TESC(I))
90    CONTINUE
      SUMA=0.
      SUMB=0.
      SUMC=0.
      SUMD=0.
      SUME=0.
      SUMF=0.
```

```
      DO 100 N=1,MAX(BT*NWP,BT*CM)
      A=1./(1000000.*N*N)
      SUMBEB=0.
      SUMBEW=0.
      SUMBEC=0.
      DO 200 JE=INBA,BT
      PJE=BESC(JE)
      IF(PJE.LE.A) GO TO 199
      SUMRB=0.
      SUMRW=0.
      SUMRC=0.
      DO 300 JR=0,JE
      CALL REFUEL(JR,JE,INTA,INTG,TESC,PREF,PJR)
      IF(PJE*PJR.LE.A) GO TO 299
      DO 400 J=0,JR
      SUM=0.
      DO 500 I=J,JR
      CALL FAD(I,JE,JR,JE*CM,L,F,PDCWL,PKWL,PRI)
      SUM=SUM+COMB(I,J)*XTOP(PBSAML,J)*XTOP(1.-PBSAML,I-J)*PRI
500   CONTINUE
      PSPEN(J)=SUM
400   CONTINUE
      DO 600 J=0,JE-JR
      SUM=0.
      DO 700 I=J,JE-JR
      CALL FAD(I,JE,JR,JE*CM,H,F,PDCWH,PKWH,PRI)
      SUM=SUM+COMB(I,J)*XTOP(PBSAMH,J)*XTOP(1.-PBSAMH,I-J)*PRI
700   CONTINUE
      PSPEN(JR+J+1)=SUM
600   CONTINUE
      SUMT=0.
      DO 800 I=0,JR*NWP
      SUM=0.
      DO 900 J=0,JR
      SUM=SUM+COMB(J*NWP,I)*XTOP(PBDL,I)*XTOP(1.-PBDL,J*NWP-I)*PS
PEN(J)
900   CONTINUE
      PL=SUM
      SUM=0.
      DO 1000 J=0,JE-JR
      SUM=SUM+COMB(J*NWP,N-I)*XTOP(PBDH,N-I)*XTOP(1.-PBDH,J*NWP-N
+I)*
     :               PSPEN(JR+J+1)
1000  CONTINUE
      PH=SUM
      SUMT=SUMT+PL*PH
800   CONTINUE
      PNW=SUMT
      IF(N.GT.JE) GO TO 298
      SUMT=0.
```

```
      DO 1100 I=0,JR
      SUM=0.
      DO 1200 J=I,JR
      SUM=SUM+COMB(J,I)*XTOP(PRECL,I)*XTOP(1.-PRECL,J-I)*PSPEN(J)
1200  CONTINUE
      PL=SUM
      SUM=0.
      DO 1300 J=0,JE-JR
      SUM=SUM+COMB(J,N-I)*XTOP(PRECH,N-I)*XTOP(1.-PRECH,J-N+I)
     :          *PSPEN(JR+J+1)
1300  CONTINUE
      PH=SUM
      SUMT=SUMT+PL*PH
1100  CONTINUE
      PNB=SUMT
      SUMRB=SUMRB+PNB*PJR
298   CONTINUE
      SUMRW=SUMRW+PNW*PJR
299   CONTINUE
300   CONTINUE
      CALL FAD(0,JE,0,JE*CM,C,F,PDCWC,PKWC,PRI)
      PCM=PCMA*PRI*PBSAMC*PTSAMC
      PNC=COMB(JE*CM,N)*XTOP(PCM,N)*XTOP(1.-PCM,JE*CM -N)
      SUMBEB=SUMBEB+SUMRB*PJE
      SUMBEW=SUMBEW+SUMRW*PJE
      SUMBEC=SUMBEC+PNC*PJE
199   CONTINUE
200   CONTINUE
      IF(N.LE.BT*NWP)
     :        THEN
              SUMA=SUMA+N*SUMBEW
              SUMB=SUMB+N*N*SUMBEW
      ENDIF
      IF(N.LE.BT)
     :        THEN
              SUMC=SUMC+N*SUMBEB
              SUMD=SUMD+N*N*SUMBEB
      ENDIF
      IF(N.LE.BT*CM)
     :        THEN
              SUME=SUME+N*SUMBEC
              SUMF=SUMF+N*N*SUMBEC
      ENDIF
100   CONTINUE
      MEANW=SUMA
      VARW=SUMB-SUMA*SUMA
      MEANBS=SUMC
      VARBS=SUMD-SUMC*SUMC
      MEANCM=SUME
      VARCM=SUMF-SUME*SUME
```

```
      PRINT'(///A,F25.16)',' MEANW=',MEANW
      PRINT'(/A,F26.16)',' VARW=',VARW
      PRINT'(/A,F24.16)',' MEANBS=',MEANBS
      PRINT'(/A,F25.16)',' VARBS=',VARBS
      PRINT'(/A,F24.16)',' MEANCM=',MEANCM
      PRINT'(/A,F25.16)',' VARCM=',VARCM
      END
      FUNCTION COMB(N,K)
      INTEGER I,K,N
      DOUBLEPRECISION C,COMB,R
      IF(K.LT.0.OR.K.GT.N) GO TO 30
      C=1.
      IF(K.LE.N-K)
     :        THEN
              DO 10 I=1,K
              R=(I+N-K)/I
              C=C*R
10            CONTINUE
              ELSE
              DO 20 I=1,N-K
              R=(I+K)/I
              C=C*R
20            CONTINUE
      ENDIF
      COMB=C
      GO TO 40
30    CONTINUE
      COMB=0.
40    CONTINUE
      END
      FUNCTION XTOP(X,P)
      INTEGER P
      DOUBLEPRECISION X,XTOP
      IF(P.LE.0)
     :        THEN
              XTOP=1.
              ELSE
              XTOP=X**P
      ENDIF
      END
      FUNCTION FP(X)
      DOUBLEPRECISION X,FP
      FP=42.*((X**6.)/6.-(X**7.)/7.)
      END
      FUNCTION FE(X)
      DOUBLEPRECISION X,FE
      FE=42.*((X**7.)/7.-(X**8.)/8.)
      END
```

```
      SUBROUTINE TIME(TMAX,TMIN,R,AIR,PET,INPG)
      INTEGER N,INPG
      REAL TMAX,TMIN,R
      DOUBLEPRECISION AIR(0:*),PET(0:*), AN,FE,FP,PT,ET
      DOUBLEPRECISION X,Y
      IF(TMAX.EQ.TMIN)
     :      THEN
            N=TMIN*R
            N=MIN(N,INPG)
            AIR(N)=1.
            PET(N)=TMIN
            ELSEIF(TMAX-TMIN.LE.1./R.AND.TMAX.NE.TMIN)
     :      THEN
            N=TMIN*R
            AN=N
            IF(N.GE.INPG) GO TO 105
            IF(AN+1.0.GE.TMAX*R)
     :            THEN
                  AIR(N)=1.
                  PET(N)=TMIN+0.75*(TMAX-TMIN)
                  ELSE
                  X=((AN+1.)/R-TMIN)/(TMAX-TMIN)
                  PT=FP(X)
                  ET=FE(X)/PT
                  AIR(N)=PT
                  PET(N)=TMIN+ET*(TMAX-TMIN)
                  X=((AN+1.)/R-TMIN)/(TMAX-TMIN)
                  PT=1.-FP(X)
                  ET=(0.75-FE(X))/PT
                  AIR(N+1)=PT
                  PET(N+1)=TMIN+ET*(TMAX-TMIN)
            ENDIF
            GO TO 205
105         CONTINUE
            AIR(INPG)=1.
            PET(INPG)=TMIN+0.75*(TMAX-TMIN)
205         CONTINUE
            ELSE
            N=TMIN*R
            AN=N
            IF(N.GE.INPG) GO TO 305
            X=((AN+1.)/R-TMIN)/(TMAX-TMIN)
            PT=FP(X)
            ET=FE(X)/PT
            AIR(N)=PT
            PET(N)=TMIN+ET*(TMAX-TMIN)
211         CONTINUE
            N=N+1
            AN=N
            IF(N.EQ.INPG) GO TO 405
            IF((AN+1.)/R.GE.TMAX) GO TO 111
            X=((AN+1.)/R-TMIN)/(TMAX-TMIN)
            Y=(AN/R-TMIN)/(TMAX-TMIN)
            PT=FP(X)-FP(Y)
            ET=(FE(X)-FE(Y))/PT
```

```
              AIR(N)=PT
              PET(N)=TMIN+ET*(TMAX-TMIN)
              GO TO 211
111           CONTINUE
              Y=(AN/R-TMIN)/(TMAX-TMIN)
              PT=1.-FP(Y)
              ET=(0.75-FE(Y))/PT
              AIR(N)=PT
              PET(N)=TMIN+ET*(TMAX-TMIN)
              GO TO 505
305           CONTINUE
              AIR(INPG)=1.
              PET(INPG)=TMIN+0.75*(TMAX-TMIN)
              GO TO 505
405           CONTINUE
              Y=(AN/R-TMIN)/(TMAX-TMIN)
              PT=1.-FP(Y)
              ET=(0.75-FE(Y))/PT
              AIR(N)=PT
              PET(N)=TMIN+ET*(TMAX-TMIN)
505           CONTINUE
        ENDIF
        END
        SUBROUTINE ESCAPE(JE,INPG,M,RL,AIR,PET,VH,VC,PROB)
        INTEGER INPG,J,JE,JR,M
        REAL RL,VH,VC
        DOUBLEPRECISION PET(0:*),AIR(0:*),PROB,COMB,XTOP,PJ
        PROB=0.
        DO 101 J=JE,INPG
        PJ=(4.*M*(RL**3.))/(3.*VH*VH*VC*(PET(J)**3.))
        IF(PJ.GT.1.0) PJ=1.0
        PROB=PROB+COMB(J,JE)*XTOP(1.-PJ,JE)*XTOP(PJ,J-JE)*AIR(J)
101     CONTINUE
        END
        SUBROUTINE REFUEL(JR,JE,INTA,INTG,TESC,PREF,PROB)
        INTEGER J,JE,JR,INTA,INTG
        REAL PREF
        DOUBLEPRECISION TESC(0:*),XPREF,COMB,XTOP,PROB
        XPREF=PREF
        IF(JE.LE.INTA)
       :      THEN
              PROB=COMB(JE,JR)*XTOP(XPREF,JR)*XTOP(1.-XPREF,JE-JR)
              ELSE
              SUMF=0.
              DO 102 J=JE-INTA,INTG
              SUMF=SUMF+TESC(J)
102           CONTINUE
              SUMS=0.
              DO 202 J=MAX(JR-INTA,0),JE-INTA-1
              SUMS=SUMS+COMB(J+INTA,JR)*XTOP(XPREF,JR)
       :             *XTOP(1.-XPREF,J+INTA-JR)*TESC(J)
202           CONTINUE
              PROB=COMB(JE,JR)*XTOP(XPREF,JR)*XTOP(1.-XPREF,JE-JR)
       :                     *SUMF+SUMS
        ENDIF
        END
```

```
      SUBROUTINE FAD(I,JE,JR,C,DUMMY,F,PDC,PK,PROB)
      INTEGER I,J,JE,JR,C,F,DUMMY,BA,BB,N
      REAL PDC,PK
      DOUBLEPRECISION PSA,PSB,PROB
      DOUBLEPRECISION COMB,XTOP
      DOUBLEPRECISION P,X,Y
      IF(JE+C.EQ.0) GO TO 1303
      N=0
      X=1.-PDC*PK
203   CONTINUE
      IF(F.LT.JE+C) GO TO 103
      F=F-JE-C
      N=N+1
      GO TO 203
103   CONTINUE
      IF(DUMMY.EQ.0)
      :      THEN
             IF(I.GT.JE-JR)
      :             THEN
                    PROB=0.
                    ELSE
                    IF(F.LT.JE-JR) GO TO 303
                    PSA=XTOP(X,N+1)
                    PROB=COMB(JE-JR,I)*XTOP(PSA,I)*XTOP(1.-PSA,JE-
R-I)
                    GO TO 403
303                 CONTINUE
                    BA=JE-JR-F
                    BB=F
                    PSA=XTOP(X,N)
                    PSB=XTOP(X,N+1)
                    P=BA*PSA/(JE-JR)+BB*PSB/(JE-JR)
                    PROB=COMB(JE-JR,I)*XTOP(P,I)*XTOP(1.-P,JE-JR-I)
403                 CONTINUE
             ENDIF
             ELSEIF(DUMMY.EQ.1)
      :      THEN
             IF(I.GT.JR)
      :             THEN
                    PROB=0.
                    ELSE
                    IF(F.GT.JE-JR) GO TO 603
                    PSA=XTOP(X,N)
                    PROB=COMB(JR,I)*XTOP(PSA,I)*XTOP(1.-PSA,JR-I)
                    GO TO 703
603                 CONTINUE
                    IF(F.GT.JE) GO TO 903
                    BA=F-JE+JR
                    BB=JE-F
                    PSA=XTOP(X,N+1)
                    PSB=XTOP(X,N)
                    P=BA*PSA/JR+BB*PSB/JR
                    PROB=COMB(JR,I)*XTOP(P,I)*XTOP(1.-P,JR-I)
                    GO TO 703
```

```
903               CONTINUE
                  PSA=XTOP(X,N+1)
                  PROB=COMB(JR,I)*XTOP(PSA,I)*XTOP(1.-PSA,JR-I)
703               CONTINUE
            ENDIF
            ELSE
            IF(F.LE.JE)
                  THEN
                  PROB=XTOP(X,N)
                  ELSE
                  Y=F-JE
                  PSA=Y/C
                  Y=JE-F+C
                  PSB=Y/C
                  PROB=PSA*XTOP(X,N+1)+PSB*XTOP(X,N)
            ENDIF
      ENDIF
      F=F+N*(JE+C)
      GO TO 1403
1303  CONTINUE
      PROB=0.
      IF(I.EQ.0) PROB=1.0
1403  CONTINUE
      END
      SUBROUTINE BOMBS(N,PK,WL,W,D,R,NSAM,PDC,PF,L,A,VB,PTSAM,PRO
B)
      INTEGER J,N,NSAM
      REAL R(0:25),PTSAM,PK,WL,W,D,PF,A,L,VB,PDC
      DOUBLEPRECISION PA,PB,PC,PD,PE,PROB,SUM
      DOUBLEPRECISION X,Y,Z,XPTSAM,XTOP
      XPTSAM=PTSAM
      SUM=0.
      DO 104 J=1,N-1
      X=1.-PK*WL*R(J)/(W*D)
      PA=XTOP(X,NSAM)
      PB=(PDC*PF*L/(PDC*PF*L-A))*EXP(-A*R(J)/VB)
      PC=(A/(PDC*PF*L-A))*EXP(-PDC*PF*L*R(J)/VB)
      PD=PA*(PB-PC)
      Y=1.-PK*WL*R(J+1)/(W*D)
      PA=XTOP(Y,NSAM)
      PB=(PDC*PF*L/(PDC*PF*L-A))*EXP(-A*R(J+1)/VB)
      PC=(A/(PDC*PF*L-A))*EXP(-PDC*PF*L*R(J+1)/VB)
      PE=PA*(PB-PC)
      SUM=SUM+J*XTOP(XPTSAM,J)*(PD-PE*XPTSAM)
104   CONTINUE
      Z=1.-PK*WL*R(N)/(W*D)
      PA=XTOP(Z,NSAM)
      PB=(PDC*PF*L/(PDC*PF*L-A))*EXP(-A*R(N)/VB)
      PC=(A/(PDC*PF*L-A))*EXP(-PDC*PF*L*R(N)/VB)
      SUM=SUM+N*XTOP(XPTSAM,N)*PA*(PB-PC)
      PROB=SUM/N
      END
*EOR
```

APPENDIX B

ESTIMATION OF THE TIME-DEPENDENT INTERCEPT DENSITY FUNCTION

This appendix describes the technique developed by R. H. Monahan of the Stanford Research Institituе for estimating λ_t, the time dependent intercept density function, for the COPEM-1 bomber penetration model. Time constraints and model scope resulted in this paper assuming a constant intercept parameter; therefore, the procedure described below is not meant to be directly applicable to this model. The procedure is presented because it gives some insight into the factors that should be considered when selecting a value of λ. It may suggest ideas for an approach to the development of a similar procedure for estimating a time-independent intercept parameter. It may also suggest a means for increasing the realism of this model by extending it to the case of a time-dependent intercept parameter.

In reviewing Monahan's derivation, one minor conceptual error was discovered. In this appendix, the method for estimating λ_t is presented as derived by Monahan; the error is pointed out and both Monahan's and the corrected results are given.

ESTIMATION OF λ_t

Recall that for a Poisson distribution with parameter λ, the waiting time to the first event is distributed exponentially with the same parameter λ. Recall also that the expected value of the exponential is $1/\lambda$. Therefore, if an estimate can be found of the expected waiting time to the first intercept of a bomber detected at time t, the

reciprocal of this will in turn be an estimate of the parameter λ_t.

To find an estimate of the waiting time to the first intercept, the penetration corridor is divided into r rows and c columns for a total number I=rc cells which are labelled sequentially from left to right and top to bottom from 1 to I as in Figure B.1.

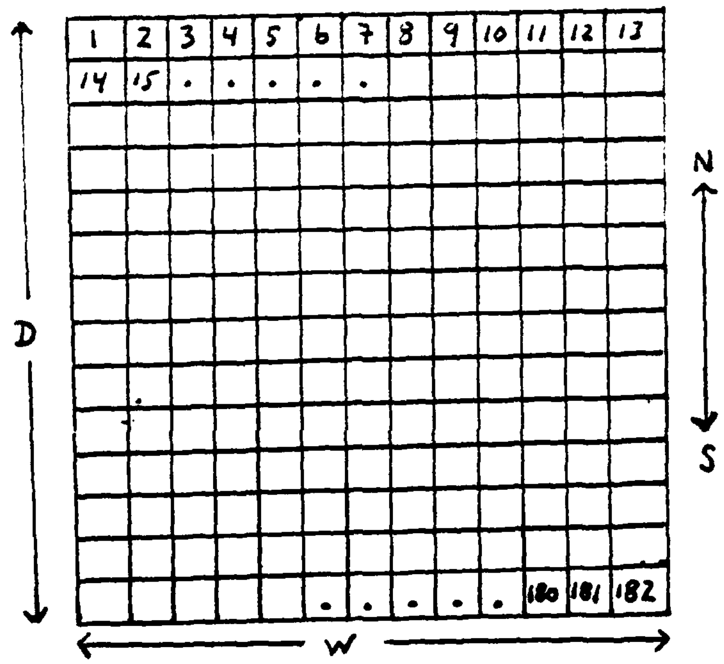

FIGURE B.1

Penetration corridor partitioned for calculation of λ_t.

Define the following variables:

n_k - the total number of fighter-interceptors at airbase k;

$n_k(t)$ - expected number of interceptors from base k that are airborne and unassigned at time t;

p_{ik} - probability that an arbitrary airborne interceptor from base k is in cell i;

$P_{ik}(t)$ - probability that there is an interceptor from base k in cell i at time t;

$q_j(t)$ - probability that there are no interceptors available to be assigned to a bomber in cell j at time t;

$q_{ij}(t)$ - probability that an interceptor assigned to a bomber in cell j at time t is from cell i;

$Q_{ij}(t)$ - probability that there is an interceptor in cell i that can be assigned to a bomber in cell j at time t;

R_{ij} - ranking of cell i relative to cell j in terms of the time it would take a fighter from cell i to intercept a bomber first detected in cell j;

T_c - bomber flight time between cells;

T_{ij} - expected time to the first intercept given that an interceptor from cell i is assigned to a bomber in cell j;

δ_{ijk} - indicator variable equalling 1 if an interceptor from airbase k in cell i is capable of intercepting a bomber in cell j and 0 if not.

Both bombers and interceptors are assumed to always be in the center of whatever cell they are occupying. Bombers fly straight down the corridor at a constant velocity while interceptors can fly in any direction at a constant velocity greater than the bombers velocity. With these assumptions, the calculation of T_{ij}, the time it takes an interceptor from cell i to intercept a bomber detected in cell j, can be calculated with basic algebra and trigonometry. This will not be done here. These T_{ij} can be used to define the R_{ij}: for a specific j, R_{ij} will equal 1 for the i which has the minimum T_{ij}, 2 for the i having the next least T_{ij}, and so on.

Now consider the $P_{ik}(t)$. For exactly $N_k(t)$ interceptors from base k airborne and unassigned at time t, one would have

$$P^*_{ik}(t) = 1-(1-p_{ik})^{N_k(t)}$$

However, it is assumed that only $E[N_k(t)] = n_k(t)$ is known so one attempts to find

$$P_{ik}(t) = E[P^*_{ik}(t)] = E[1-(1-p_{ik})^{N_k(t)}] \tag{B.1}$$

If $N_k(t)$ is assumed to be binomially distributed with parameters $p=n_k(t)/n_k$ and $N=n_k$, then it can be shown that

$$P_{ik}(t) = 1-[1-p_{ik}n_k(t)/n_k]^{n_k} \tag{B.2}$$

With this result, one can now find the probability that there is an interceptor in cell i that can be assigned to a bomber detected in cell j at time t;

$$Q_{ij}(t) = 1-\prod_{k=1}^{K}[1-\delta_{ijk}P_{ik}(t)], \tag{B.3}$$

where K is the total number of interceptor bases.

To continue, let i_m denote the value of i for which $R_{ij}=m$ for a specific j. One can now find the probability that the interceptor assigned to a bomber in cell j is from cell i (Note: it is possible that more than one interceptor will be assigned to a bomber in cell j at time t but one is only concerned here with the interceptor which will make the earliest interception of the bomber.)

$$q_{i_mj}(t) = [1-\sum_{x=1}^{m-1}q_{i_xj}(t)]Q_{i_mj}(t), \text{ with } q_{i_1j}(t)=Q_{i_1j}(t) \tag{B.4}$$

From this, the probability that there are no interceptors available to be assigned to a bomber in cell j at time t is

$$q_j(t) = 1 - \sum_{x=1}^{I} q_{i_x j}(t) \qquad \text{B.5}$$

Now consider the expected time to the first intercept of a bomber detected in cell j at time t. This value may be broken into two parts and expressed

$$E[T_j(t)] = E_1[T_j(t)] + E_2[T_j(t)] \qquad \text{B.6}$$

where one has conditioned on the availability or unavailability of an interceptor to be assigned to the bomber. The first term represents the expected value of the time to be first intercept given that there is an interceptor available to be assigned to the bomber times the probability that there is an interceptor available. The second term represents the expected time to the first intercept given that there are no interceptors immediately available to be assigned to the bomber times the probability that there are no interceptors available. If A is the event that there is an interceptor available to be assigned to a bomber detected in cell j at time t, then the above can be expressed mathematically as

$$E_1[T_j(t)] = E[T_j(t)|A]P(A) \quad \text{and} \qquad \text{B.7a}$$

$$E_2[T_j(t)] = E[T_j(t)|A^c]P(A^c) \qquad \text{B.7b}$$

(Note: the E_1 and E_2 notation may seem a litte confusing but is being used here because that is how the derivation is presented in Ref 5.)

The expected value of the time to the first intercept if there is an interceptor immediately available can be expressed in terms of quantities already expressed,

$$E_1[T_j(t)] = \sum_{i=1}^{I} q_{ij}(t)T_{ij} \qquad \text{B.8}$$

It remains to find the expected value of the time to the first intercept if there are no interceptors available to be assigned to the bomber detected in cell j at time t.

To approximate this quantity, Monachan suggests the following appraoch. Allow the bomber to continue to the next cell, consuming a time T_c. Attempt to assign an interceptor. If there are still none available, allow the bomber to continue to the next cell, repeating until a time is reached when an interceptor is available to be assigned to the bomber. To find the expected time to intercept if there are no interceptors available at t, one can condition on the number of cells the bomber flies before an interceptor becomes available to be assigned to it. The following expression is proposed:

$$\begin{aligned} E_2[T_j(t)] &= q_j(t)[1-q_j(t)][T_c+E_1(T_j(t))] \\ &+ q_j(t)^2[1-q_j(t)][2T_c+E_1(T_j(t))] \\ &+ \ldots \\ &+ q_j(t)^m[1-q_j(t)][mT_c+E_1(T_j(t))] \\ &+ \ldots \end{aligned} \qquad \text{B.8}$$

Several things should be noted. First, both the T_j and the q_j may be functions of time. Furthermore, each time the bomber moves

to a new cell, the value of j is incremented by c, the number of columns. With these two observations, the second term in the above sum should look like

$$q_j(t)q_{j+c}(t+T_c)[1-q_{j+wc}(t+T_c)][2T_c+E_1(T_{j+2c}(t+2T_c))] \qquad B.9$$

and the other terms would be modified similarly. Monahan's only comment on the approximation is that he "assumes a semblance of stationarity (Ref 5:46)."

The second thing to be noted is that there are only a finite number of cells and if the bomber penetrated beyond the end of the corridor he would presumably be safe from interception. The above sum, therefore should be finite, rather than infinite. No comment is made on this.

There is nothing really wrong with these two approximations as long as they are recognized as such. However, even if one accepts the approximations, there is an error in the sum given in Equation B.8.

Recall that, using A as previously defined,

$$E_1[T_j(t)] = E[T_j(t)|A]P(A) \qquad B.10$$

Rearranging the above expression and noting that P(A) can be expressed as $1-q_j(t)$, one has

$$E[T_j(t)|A] = E_1[T_j(t)]/(1-q_j(t)) \qquad B.11$$

Now consider what is happening when E_2 is conditioned over the number of cells the bomber flies before an interceptor becomes avail-

able to be assigned to it. The m^{th} term of the sum gives the expected waiting time for the first intercept given that an interceptor has become available after the bomber has flown exactly m cells times the probability that the first interceptor becomes available at that time. The probability is geometric and is given by the expression

$$P(B_m) = q_j(t)^m[1-q_j(t)] \qquad B.12$$

where the approximations discussed above are being used.

The expected waiting time given that an interceptor becomes available after the bomber has flown m cells is just the time it took the bomber to fly the m cells plus the expected time to the first interceptor <u>given that there is an interceptor available to be assigned</u>.

$$E(W_m) = mT_c + E[T_j(t)|A] \qquad B.13$$

(In the above expressions, B_m is the event that the first interceptor to be assigned to the bomber becomes available after the bomber has flown exactly m cells and W_m is the waiting time to the first intercept given B_m.)

The m^{th} term of the sum for E_2 should therefore read:

$$m^{th} \text{ term} = P(B_m)E(W_m)$$

$$= q_j(t)^m[1-q_j(t)][mT_c + E(T_j(t)|A]$$

$$= q_j(t)^m[1-q_j(t)][mT_c + E_1(T_j(t))/(1-q_j(t))] \qquad B.14$$

This differs from the expression in Equation B.8 by the factor of $1-q_j(t)$ in the denominator of E_1.

The sum can easily be reduced by using the identities

$$\sum_{i=1}^{\infty} x^i = x/(1-x) \quad \text{and}$$

$$\sum_{i=1}^{\infty} ix^i = x/(1-x)^2$$

Monahan finds

$$E_2[T_j(t)] = q_j(t)E_1[T_j(t)] + T_c q_j(t)/[1-q_j(t)] \qquad B.15$$

which can be added to E_1 to give

$$E[T_j(t)] = [1+q_j(t)]E_1[T_j(t)] + T_c q_j(t)/[1-q_j(t)] \qquad B.16$$

The corrected expressions are

$$E_2[T_j(t)] = q_j(t)E_1[T_j(t)]/[1-q_j(t)] + T_c q_j(t)/[1-q_j(t)] \qquad B.17$$

and

$$E[T_j(t)] = E_1[T_j(t)]/[1-q_j(t)] + T_c q_j(t)/[1-q_j(t)] \qquad B.18$$

Because of the assumption of "semblance of stationarity," an easy method exists to check the above expression. If one conditions the expected time to first intercept on the availability or unavailability of an interceptor to be assigned to the bomber, one has

$$E[T_j(t)] = E_1[T_j(t)] + q_j[T_c + E(T_j(t))] \qquad B.19$$

This is a simple linear equation $E[T_j(t)]$ which can be solved to give

$$E[T_j(t)] = E_1[T_j(t)]/[1-q_j(t)] + T_c q_j(t)/[1-q_j(t)] \qquad B.20$$

as above.

To conclude the original problem, which was to estimate the parameter λ_t, one has

$$E_t(T) = \sum_{j=1}^{I} P_j(t)E[T_j(t)] \qquad \text{B.21}$$

where T is the time to the first intercept and $P_j(t)$ is the probability that a bomber detected at time t is in cell j. The reciprocal of $E_t(t)$ is the desired estimate of λ_t.

ESTIMATION OF $n_k(t)$

The next problem is to find a suitable estimate for $n_k(t)$, the expected number of fighters from airbase k available for assignment as a function of time. Define the following variables:

Δt - interval between successive estimations of λ_t;

T_i - i^{th} time interval for the calculation of λ_t; T_i is the time interval $[(i-1)\Delta t, i\Delta t]$;

n_k - total number of fighters stationed at base k;

$N_d(T_i)$ - the number of bombers detected in the time interval T_i;

$N_k(T_i)$ - the number of fighters from base k assigned to bombers detected in the time interval T_i;

$P_k(T_i)$ - the probability that a bomber detected in the time interval T_i will have a fighter from base k assigned to it;

T_f - total time a fighter can remain airborne before recycling;

T_r - the minimum time for a fighter to recycle;

r_s - the scramble rate at which fighters take off, measured in fighters/unit time.

The approach is to first construct a fighter availability curve for base k assuming that no bombers are detected or intercepted. The curve is then iteratively modified at discrete time intervals to account for the change in fighter availability arising from the detection and interception of bombers in each interval.

Assume that before the battle, the number of airborne fighters is maintained at some constant level, $n_k(0)$. The maximum possible value for that constant, $n_k^*(0)$, is

$$n_k^*(0) = \frac{T_f}{T_f+T_r} n_k \tag{B.22}$$

At time t=0, all the fighters are flushed and the number of airborne fighters increases linearly according to the scramble rate, r_s. All the fighters will be airborne at time t_1,

$$t_1 = \frac{n_k-n_k(0)}{r_s} \tag{B.23}$$

At some time t_2, airborne aircraft will begin to recycle and the number of airborne fighters will now decrease according to the scramble rate for a length of time T_r, the time taken by one fighter to recycle. At this time, the first fighters to return for recycling will be taking off again and the airborne force level will remain constant until the last fighter to be recycled has landed. The airborne force level will then begin to increase again according to the scramble rate and the process will be repeated. The process is illustrated in Figure B.2 for $n_k(0)$=20, n_k=30, r_s=1/minute, T_f-50 minutes, and T_r=20 minutes.

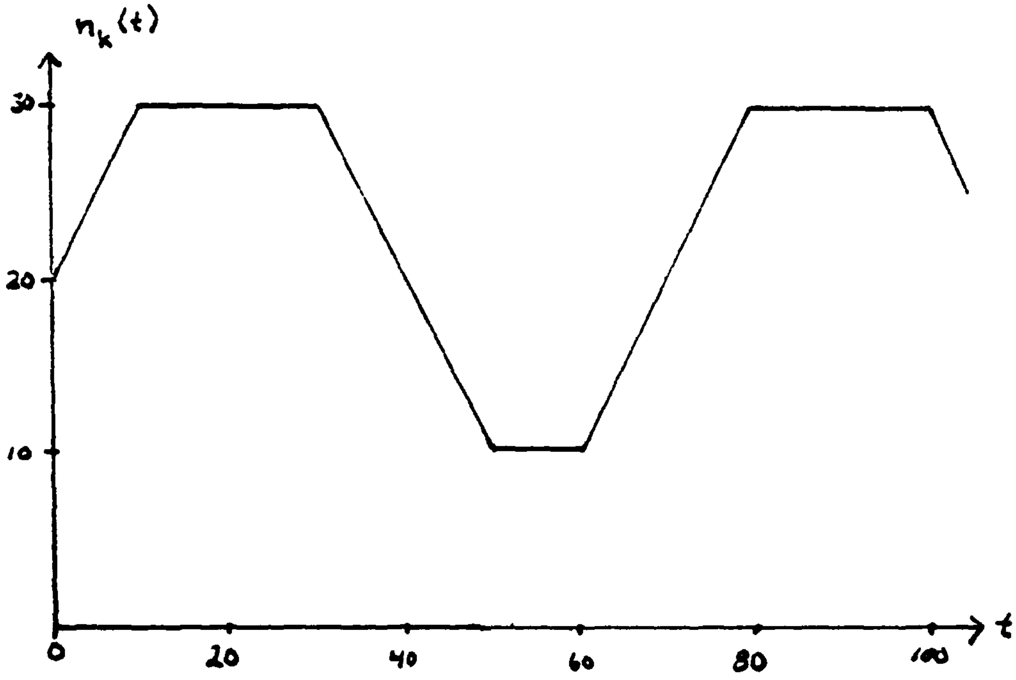

FIGURE B.2

Airborne force level for one base.

Figure B.2 gives the number of airborne fighters from base k as a function of time. To find the number of airborne fighters available for assignment, it is necessary to subtract off the number of fighters assigned as a function of time. The method for doing this will be demonstrated by continuing the example presented in Figure B.2. For this example, esimates (λ_{T_i}) will be computed afor the intercept density function λ_t, at 10-minute intervals. An estimate (user specified of λ_0 is assumed to be such that λ_0^{-1}, the expected time to first intercept, is 14 minutes.

Using a convolution of the detection time and arrival time distributions ($p_d(t_d)$ and $p_a(t_a)$), an expected number of bombers detected, denoted $N_d(T_1)$, can be found for the first 10-minute interval. Say this is 7 bombers. It is assumed that when making an original estimate of λ_0, an unconditional probability, $P_k(T_1)$, that a bomber detected in this time interval will have a fighter from airbase k assigned to it can be calculated. For this example, this probability is assumed to be 5/7. From this, one finds the expected number of fighters from airbase k assigned to attack bombers in this interval is

$$N_k(T_1) = P_k(T_1)N_d(T_1) \qquad \text{B.24}$$

which in this example is 5.

Thus at time t=10 minutes, the number of fighters from base k available for intercept has decreased by 5. This point is labelled A in Figure B.3. It is assumed for this example that the probability of kill, given an intercept, for a fighter against a bomber is 0.7. The expected time to a successful intercept (when the bomber is killed) is, therefore, equal to 14/0.7, or 20 minutes. Thus at time t=20 minutes, all the fighters assigned to bombers in the first 10-minute interval will rejoin the unassigned fighter force. This is indicated by the line segment connecting points B and C in Figure B.3.

This completes the modification to the availability curve based on the activity during the first 10-minute interval; the remainder of the curve remains unchanged from Figure B.2. At this point, λ_{T_i} can be calculated.

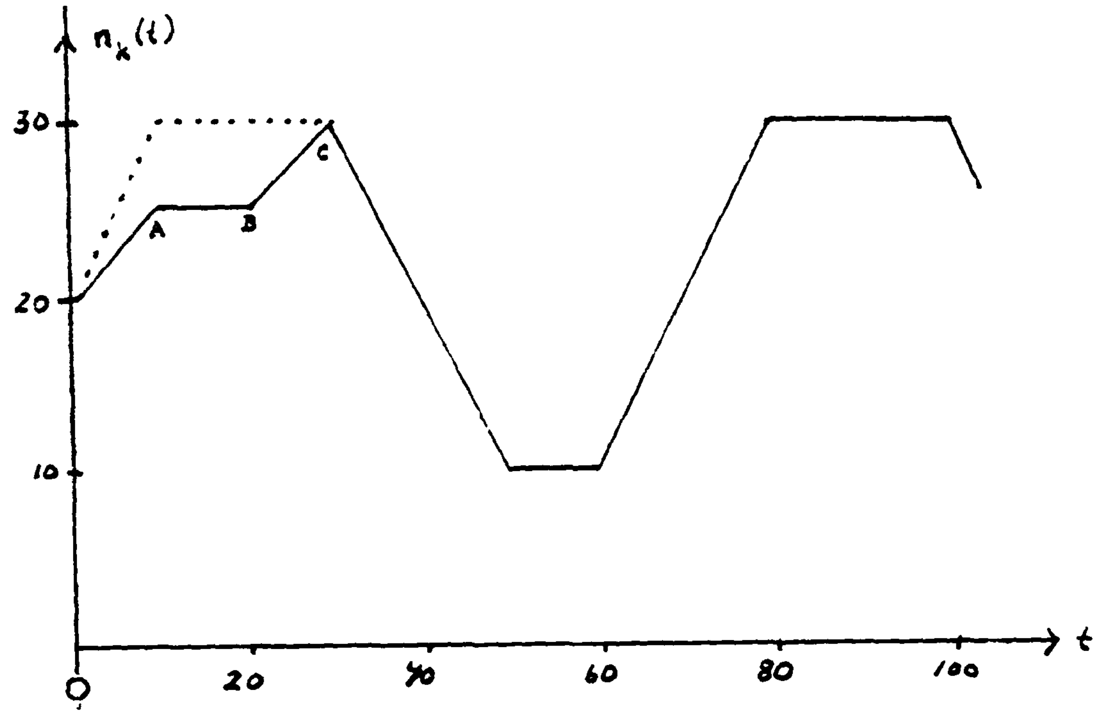

FIGURE B.3

Fighter availability curve modified for activity in T_1.

This procedure is repeated for T_2, T_3, ... T_n. At each stage, the previous estimate of the intercept density function ($\lambda_{T_{i-1}}$) is used to find the expected time to a successful intercept. The availability curve can then be modified for the activity in that time interval and a value for λ_{T_i} can be calculated.

In performing the above calculations, it has been assumed that λ_{T_i} was constant over each interval. Specifically, $\lambda_t = \lambda_{T_{i-1}}$ for the entire i^{th} interval. However, once the calculations are complete, and estimates have been made for all the λ_{T_i}, values for λ_t at interior points on the intervals may be found by interpolation if desired. This completes the derivation.

VITA

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This paper describes an analytic model of the strategic bomber penetration mission. The model covers all parts of the mission from take off to kill or recovery. Specifically, the model includes representations of base escape, aerial refueling, forward air defense, barrier SAMs, area fighter-interceptor defense, random area SAMs, terminal SAMs, weapons delivered and recovery. Cruise missiles are included in the model.

For each part of the mission listed above, a distribution is derived for the number of bombers that survive given the number that begin that stage of the mission. A computer program has been written to convolute these distributions to find distributions for the total number of bombers that survive, the total number of weapons released, and the total number of cruise missiles that destroy targets.

One major advantage of this model is its scope. It includes descriptions of all the events that occur during the mission which will have an impact on the outcomes of later parts of the mission.

A second major advantage of this model is the inclusion of calculations for the variances of the results of the model. This is an improvement on other analytic models which give only expected value results and hence contain no indication of the possible variability of those results.

All the mathematical derivations for this model are contained in this paper. The paper also includes some initial results from the computer program which demonstrate some of the many outputs that can be obtained from the model.